Optimal managing of forest composition using data simulated optimal control

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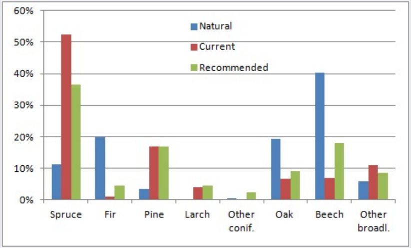
Outline

Forestry situation in the Czech Republic

- 33 % of the Czech country forested
- economic importance for rural communities
- replacing natural ecosystems by *spruce*
- lack of biodiversity
- soil damage

Motivation

Composition of tree species in the Czech Republic



- need for forest composition modification
- legislative and subsidy measures
- unknown long run impacts

Research aims

1. to model long run impacts of current measures via dynamic optimization
2. to estimate valid underlying functions
3. to complete the data through simulation
4. to develop the model in two stages
   - coarse version ("average" characteristics)
   - fine version (local specific characteristics)

Outline

The simple optimal control problem

$$
\begin{aligned}
\max \quad & V = \int_0^T \Pi(x,u,t) e^{-\rho t} \mathrm{d}t && (1)\\
\text{s.t.} \quad & \\
& \dot{x} = f(t,x,u), && (2)\\
& x(0) = A \ \text{given}, \quad x(T) = \text{free} && (3)\\
\text{and} \quad & \text{additional constraints}
\end{aligned}
$$

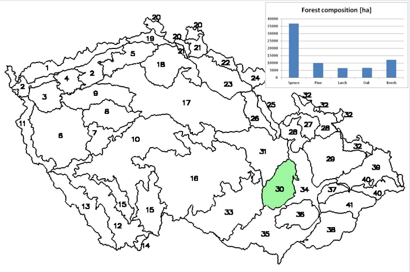
V objective functional ($\Pi(t)$ - total profit)

$x_i(t)$ state variables

$u_i(t)$ control variables

The Drahanska highlands case

Area localization in the Czech Republic



- 72122 ha of forest land
- legislative: min 25 % reforested by soil improving species (SIS)
- subsidies for:
  - seedlings (oak, beech),
  - intensive reforesting with SIS (above 30 %)

Assumptions and restrictions

- not consider forestation of non-forest land
- substituting other species for spruce: only area of spruce may decrease in time
- changing forest composition after clear cutting spruce at rotation age
- current profit maximization:
  - revenues (timber selling, subsidies)
  - costs (planting, forest stand care, harvesting)

The elements of optimization model

- state variables x_{it} - area forested by tree species i in time t, $t = 1, \ldots, T$
- control variable u_{it} - change in total area forested by species i in year t
- equation of motion $x_{i(t+1)} = x_{it} + u_{it}$
- objective function

$$F = \sum_{t=1}^{T} \delta^t \left[\sum_{i=1}^{n} (G_{it} - K_{it} + \sigma_i u_{it}) + \sum_{k=1}^{5} \rho_k w_{kt} \right]$$

$$\begin{aligned} G_{it} &= g_{i0} + g_{i1} x_{it} + \frac{1}{2} g_{i2} x_{it}^2 \\ K_{it} &= k_i + a_i x_{it} + b_{i1} u_{it} + \frac{1}{2} b_{i2} u_{it}^2 \end{aligned}$$

The complete dynamic optimization model

$$\max_{x_{it}, u_{it}, w_{kt}} \quad F = \sum_{t=1}^{T} \delta^t \left[\sum_{i=1}^{n} (G_{it} - K_{it} + \sigma_i u_{it}) + \sum_{k=1}^{5} \rho_k w_{kt} \right]$$

$$\text{s.t.} \quad x_{i(t+1)} = x_{it} + u_{i(t+1)}$$

$$\sum_{i=1}^{n} x_{it} = A \text{ for all } t$$

$$\sum_{j=1}^{4} u_{jt} \leq x_{5(t-1)}/\tau_5$$

$$u_{1t} + u_{2t} + u_{4t} \geq 0.25\, x_{5(t-1)}/\tau_5$$

$$\sum_{k=1}^{5} \rho_k w_{kt} = u_{1t} + u_{2t} + u_{4t}$$

$$\sum_{k=1}^{5} w_{kt} = x_{5t}/\tau_5$$

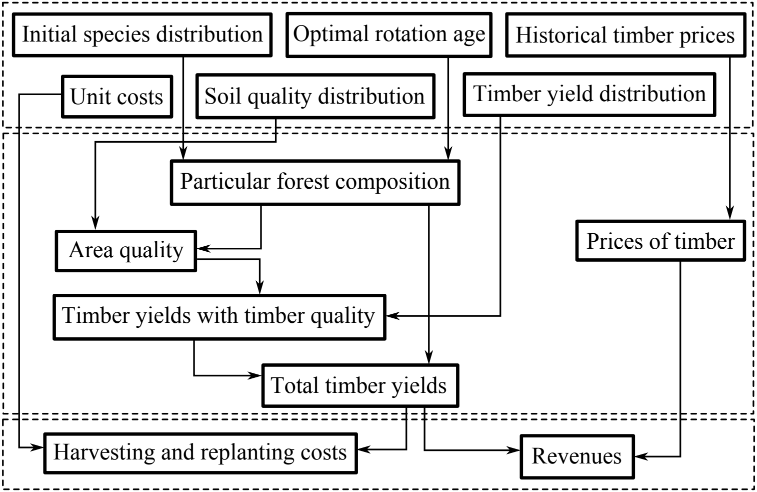
$$\text{and} \quad x_{i0} \text{ given}, \ x_{iT} \text{free}, \ x_{it}, \ u_{jt} \geq 0, \ 1 \leq i, k \leq 5, , \ 1 \leq j \leq 4.$$

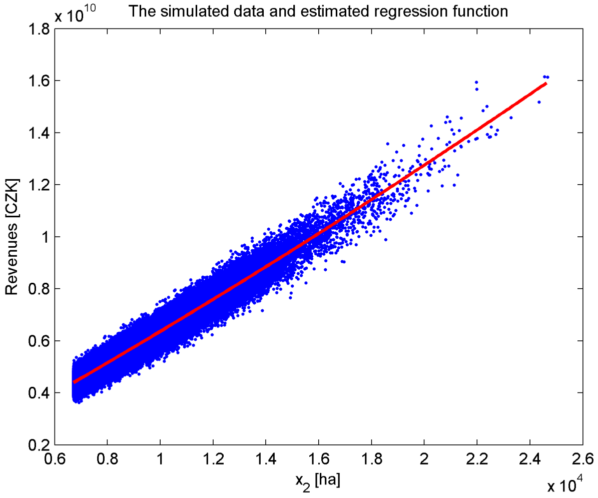
Estimating revenue and cost functions

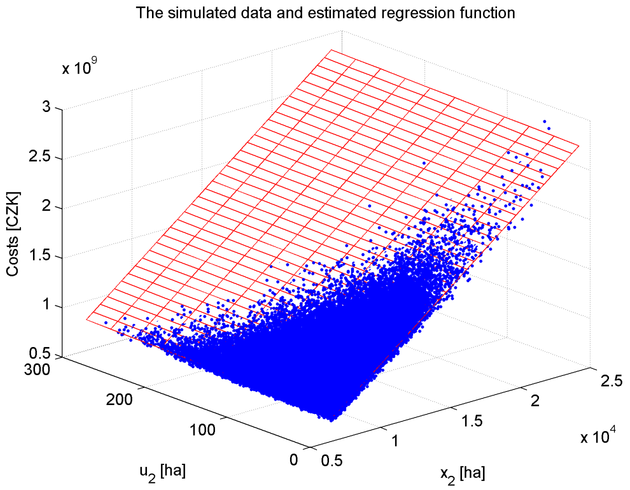
$$
\begin{aligned}
G_{it} &= g_{i0} + g_{i1} x_{it} + \frac{1}{2} g_{i2} x_{it}^2 \\
K_{it} &= k_i + a_i x_{it} + b_{i1} u_{it} + \frac{1}{2} b_{i2} u_{it}^2
\end{aligned}
$$

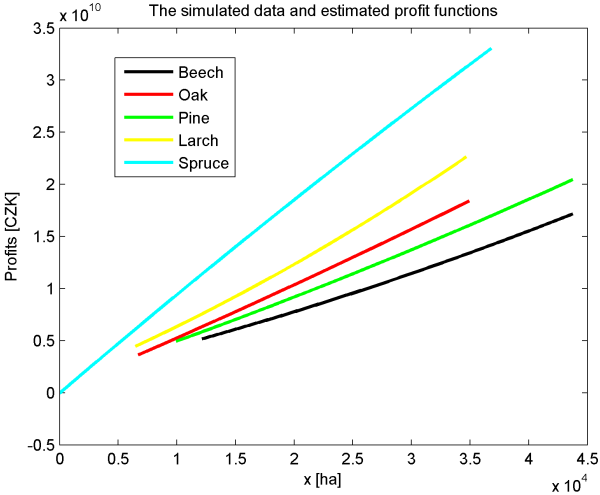
- real data on environmental, technological and economical characteristics from public sources and Mendel university forest enterprise.
- in original form not sufficient for estimation of the cost and revenue functions parameters.
- it is possible to obtain such estimates using extensive simulation.
- totally 100,000 combinations of necessary values were simulated.

Outline


Initial species distribution
Optimal rotation age
Historical timber prices
Unit costs
Soil quality distribution
Timber yield distribution
Particular forest composition
Area quality
Prices of timber
Timber yields with timber quality
Total timber yields
Harvesting and replanting costs
Revenues

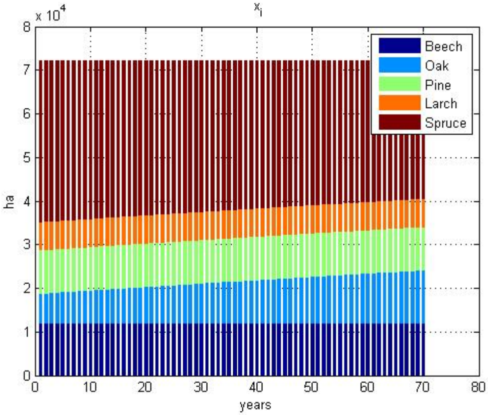

The simulated data and estimated regression function
x 10^10
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
Revenues [CZK]
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.2
2.4
2.6
x_2 [ha]
x 10^4


The simulated data and estimated regression function
x 10^9
3
2.5
2
1.5
1
0.5
Costs [CZK]
300
200
100
0
u2 [ha]
0.5
1
1.5
2
2.5
x 10^4
x2 [ha]


The simulated data and estimated profit functions
x 10^10
Profits [CZK]
x [ha]
x 10^4
Beech
Oak
Pine
Larch
Spruce
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5

Outline

Time path for forest composition



- replacing spruce by oak: by minimal admissible speed
- different long run subsidy scenarios: no effect

Conclusion and future work

- revenue and cost functions: realistic representation
- first stage (coarse) model: results correspond to the observed situation
- second stage model:
  - incorporate fine structure of the Drahanska highlands area
  - simulation based on particular local characteristics (terrain profile, over sea altitude, soil quality,...)
- subsidy policy optimization

References

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Thank you.