Nonlinear Optimization of the Indemnity Level under Drought Hazard and Basis Risk

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Abstract

Index-based insurance is thought to be efficient alternative risk management against the impacts of natural hazards, especially for the drought. The most powerful policy is the one that directly based on the positive relation between any suitable index measure and the yield loss. At this step, the insurer must be aware of the basis risk phenomena coming from the mismatch between the real and the expected loss of the policyholder. For this reason, the modelling part of this type of insurance is the primary step to manage the loss resulted in any dry season. We propose alternative models to describe the wheat yield loss for the selected farms in Turkey using MCMC method. Based on these models, actuarial valuation is made for each plantation area. Nonlinear optimization method is considered for the basis risk estimation of index-based insurance contracts.

Keywords: MCMC, drought insurance, basis risk, nonlinear optimization.

1 Introduction

Agriculture is a weather dependent sector in the economy [1]. Especially, with the increasing trend on climate change, food production will be decreased in the near future for many countries. In recent years, Turkey faced with dry seasons and it is predicted that such conditions will continue [2]. Nevertheless, the insurance system in Turkey is recently evolving to cover agricultural loss. In 2005, Agricultural Insurance Pool (TARSIM) was established to provide coverage for the risks threatening the agricultural industry. One of the main aims of TARSIM is providing loss coverage against the catastrophe risks. They provide a crop insurance in Turkey loss arising from hail, storm, fire, tornado, flood, landslide and earthquake with a 50% government subsidization. However, there is no coverage for the drought risk even if the farmers union demands such type of protection [3]. In this sense, the drought coverage is still a controversial topic in Turkey. The first index-based insurance contract is proposed for the provinces of Central Anatolia in 2012 [5].

This study aims testing the basis risk performance of different index based insurances in case of drought hazard in Turkey. Rainfed wheat productions in the 11 TIGEM stations are considered, which are located mostly at Central Anatolia. We first use the crop and location specific model developed by FAO to simulate the water related variables such as evapotranspiration, water deficiency, water satisfaction index, and estimate the crop yield function for rainfed wheat production in selected stations. A spatio-temporal yield model is estimated by Bayesian method through the use of Markov Chain Monte Carlo (MCMC) algorithms. Standardizing the simulated variables over Normalised Difference Vegetation Index (NDVI), the impact of drought related variables on wheat yield is studied. We use the Fixed Effect Spatio-Temporal model to predict the wheat yield for the selected 11 stations. Based on these estimations, we design a one-year insurance policy for year 2006. In order to compare the basis risk performance of given insurance contracts, we calculate the premium and indemnity payments for the selected farms.

As mentioned above, basis risk is the most important limitation of index-based insurance [6]. Mainly, the efficiency and feasibility of any policy are directly based on the basis risk minimization. In general, there exist three types of basis risk in the literature called spatial, temporal and product. The effectiveness of index-based insurance policies is still a controversial issue because of the basis risk problem. In this study, we concentrate on the product type basis risk which occurs when there is pure relationship between the selected index measure and the crop yield [7]. For this reason, we develope a simple ratio to compare the basis risk reduction power of any designed policy. It gives a hint about the product type basis risk arose from the Fixed Effect Spatio-Temporal (FEST) model which is used for predicting wheat yield loss. However, all basis risk types should be considered simultaneously while designing any index-based insurance contract [8].

The rest of the paper starts with the model selection for the wheat yield estimation for the selected farm stations. This part is followed by the insurance contract properties. After that, the nonlinear indemnity optimization problem and its results are given. The paper concludes with some comments on the findings from the proposed approach.

2 Model Selection

To design any index-based contract and perform actuarial calculations, wheat yield estimation is required. Additionally, linear models allow the researcher to derive faster and straightforward results. For these reasons, using the selected index measures, province based linear regression and fixed effect (FE) panel models are mostly implemented. Moreover, the bayesian approach is utilized to increase the efficiency of the prediction models. Bayesian Spatio Temporal models are employed to obtain prediction functions for the wheat yield. Besides, the prediction functions are essential in order to analyze the relationship between the wheat yield and the predictor variables. According to Yıldırak et al.[4], (FEST) model is preferable to predict the wheat yield for year 2006 at 11 stations. Under FEST model, the most powerful explanatory variables are determined as $ETAVERT_i$ and $WDEFVERT_{fr}$. For the detailed information about these predictors, please see [4].

Bayesian FEST model has some advantages while designing index-based insurance. Firstly, it gives more accurate estimation when the data set is limited. Moreover, such models are useful to reduce the basis risk problem resulting from unobserved heterogeneity.

For the illustration, we consider one of the TIGEM stations, named as Anadolu Farm. The wheat yield is estimated based on the linear model below [4]:

$$log(Y_{i,t}) = s_i + u_t + (-0.09).(ETAVERT_i)_{i,t} + (-0.35).(WDEFVERT_{fr})_{i,t} \quad (1)$$

where $Y_{i,t}$ denotes wheat yield, s_i and u_t are spatial and temporal random effects for each local farm respectively. Temporal effect for the year 2006 is calculated as follows [4]:

$$u_{2006} = 1.54.(1 - 0.47) + 0.47.(1.61) = 1.57, \tag{2}$$

and s_i=4.31 is fixed for the selected farm and independent of time. Thus, we derive the predicted wheat yield in 2006 for given $ETAVERT_i = 0.77$ and $WDEFVERT_{fr}$ = 1.52 as follows:

$$Y_{i,2006} = 4.31 + 1.57 + (-0.09).(0.77) + (-0.35).(1.52) = 1.97 ton/ha. \tag{3}$$

Similarly, the wheat yield predictions are made for each TIGEM farm for the period concerned [4]. These yield estimations for each farm will be the base of calculation of pure premium in insurance design.

3 Pure Premium Design

A threshold based insurance contract for year 2006 is constructed according to the prediction model proposed above. We use the estimation models to set a strike level for the selected explanatory variables. Morever, premium and indemnity calculations are made based on this trigger level for 11 stations. We test the basis risk performance of insurance policies for all farms.

For the contract year, the wheat yield is estimated using classical Box-Jenkins method treating wheat yield as a time series data. This prediction for the year 2006 is made based on the best Box-Jenkins process. This value is used to make inference about the yield loss expectation.

For the index-based insurance contracts, Pure Premium (PP) is calculated by:

$$\text{PP} = E[X] = E[Losses] = (\frac{1}{n}) \sum_{i=1}^{n} \widehat{I_i} \tag{4}$$

where n is the number of years and $\widehat{I_i}$ represents the claim payment of the index based insurance starting from year 1 to n according to the FEST model. The PP value is summarized in Table 1 where AAYL represents the Average Annual Yield Loss. As a result of high value of AAYL, for the farms Konuklar and Kocaş, the PP amount results are challenging. Indeed, just PP amount seems that it is not bearable to buy insurance by the individual farmers after adding other expenses.

It indicates similar to other developing countries, the importance of governmental subsidization in Turkey for designing any index-based insurance policy.

Table 1: PP results for TIGEM Farms in Turkey

Local farm	AAYL (ton/ha)	PP (TL/ha)
Anadolu	0.14	51.10
Gökhöyük	0.24	79.90
Ceylanpınar	0.24	83.93
Bala	0.17	59.14
Altınova	0.18	62.50
Malya	0.19	72.94
Gözlü	0.17	58.76
Konuklar	0.31	106.58
Kocaş	0.31	109.94
Polatlı	0.20	72.59
Ulaş	0.17	61.22

The indemnification, denoted by (I), is calculated based on the following:

$$\text{I} = I(X_1, X_2) = (w).\gamma_1.\max(X_1 - S_1, 0) + (1 - w).\gamma_2.\max(X_2 - S_2, 0) \tag{5}$$

where, $I(X_1, X_2)$ denotes the claim amount that will be paid to the insured in 2006, w is the weight coefficient, S_1 and S_2 represent the strike levels for the selected index variables, X_1 and X_2 are the observed values of $ETAVERT_i$ and $WDEFVERT_{fr}$ in 2006 respectively, γ_1 and γ_2 define the size of the index level that quantifies the indemnity payment, i. e. thick size value.

As we mentioned above, to test the basis risk performance of each contract, the Basis Risk Reduction Power (BRRP) ratio is generated and used for each TIGEM Farm as:

$$\text{BRRP} = I(X_1, X_2)/ELOP \tag{6}$$

where ELOP, the expected provincial wheat yield loss, defined as:

$$\text{ELOP} = WP \times max(FWY - OWY, 0). \tag{7}$$

Here, WP is the wheat price belonging to the last year before the contract expiry, FWY and OWY represent the forecasted and observed wheat yield of the contract year respectively. If BRRP ratio approaches to the value 1, the designed policy gets the lowest basis risk.

4 Optimum Indemnity Level

Under FEST model, we consider two predictors in a single equation for the yield estimation. For this reason, we need to determine the strike levels S_1 and S_2 for the selected index measures. However, this calculation is not straightforward compared to the case defined in [5]. Besides, there is numerical difficulty to compute the thick size defined in (5) because, it is a function of strike levels [5]. For this reason, an optimization setup with feasible constraints is employed.

For simplicity, the nonlinear optimization method is considered to deal with the troublesome calculations of indemnification. It is assumed that the strike levels and thick sizes are unknown at first and indemnity amount is function of these parameters. The problem is formulized, in general, as follows ;

$$\begin{aligned} &\textbf{Maximize and Minimize } I(S_i, \gamma_i) \\ &\quad \text{subject to constraints ;} \\ &\quad\quad a_i \leq S_i \leq b_i, \\ &\quad\quad c_i \leq \gamma_i \leq d_i, \end{aligned}$$

for i=1,2 and predefined boundary values a_i, b_i, c_i and d_i to find the possible highest and smallest indemnity values.

Firstly, it is assumed that, based on the given definition of indemnity in (5), both predictors have the same contribution to the indemnity so that we set $w = 0.5$. Under this nonlinear optimization setup, there are four different scenarios based on the relation between observed values and strike levels of corresponding predictor. We examine the optimization of $I(S_1, S_2, \gamma_1, \gamma_2)$ by using these different cases:

Case 1: Let $S_1 < X_1$ and $S_2 < X_2$. In this case, (5) is transformed to a nonlinear function of γ_i and S_i for i=1,2. Then;

$$\begin{gathered} \text{Max and Min } I(S_1, S_2, \gamma_1, \gamma_2) \\ \text{subject to ;} \\ S_1 < X_1 \end{gathered}$$

$$S_2 < X_2$$
$$\gamma_1 \geq 0$$
$$\gamma_2 \geq 0$$

Case 2: Let $S_1 < X_1$ and $S_2 > X_2$. In this case, there is no payout because of the second predictor . Thus, $I(S_1, S_2, \gamma_1, \gamma_2)$ is just function of γ_1 and S_1. Then;

$$\text{Max and Min } I(S_1, \gamma_1)$$
$$\text{subject to ;}$$
$$S_1 < X_1$$
$$\gamma_1 \geq 0$$

Case 3: Let $S_1 > X_1$ and $S_2 < X_2$. In this case, there is no payout because of the first predictor . Thus, $I(S_1, S_2, \gamma_1, \gamma_2)$ is just function of γ_2 and S_2. Then;

$$\text{Max and Min } I(S_2, \gamma_2)$$
$$\text{subject to ;}$$
$$S_2 < X_2$$
$$\gamma_2 \geq 0$$

Case 4: Let $S_1 > X_1$ and $S_2 > X_2$. In this case, based on (5), there is no indemnification since $I(S_1, S_2, \gamma_1, \gamma_2)$=0 directly.

The above mentioned optimization problem is implemented in Matlab 2009 environment with the "optimtool". The procedure is exemplified by using the result of Anadolu Farm. The derived $I(S_1, S_2, \gamma_1, \gamma_2)$ function is maximized and minimized to understand the possible lower and upper bounds for the indemnity. We assume that both predictors fall below the observed values and consider the Case 1 to derive nontrivial solution to the proposed optimization problem.

By using the Matlab optimization interface, we determine the minimized value of $I(S_1, S_2, \gamma_1, \gamma_2)$ and $-I(S_1, S_2, \gamma_1, \gamma_2)$ to generate a lower and upper bound of the indemnification. Moreover, the correspongding values of γ_i and S_i are listed. Because of the importance of loss ratio, we assume that $I(S_1, S_2, \gamma_1, \gamma_2)$ can be minimum as the PP and maximum as the 5 times PP given in (4). Furthermore, we set 50 as a lower bound and 100 as an upper bound for γ_i values [5]. Table 2 presents the results for the station Anadolu Farm.

Table 2: Optimization result summary for Anadolu Farm

$I(S_1, S_2, \gamma_1, \gamma_2)_{min}$ $I(S_1, S_2, \gamma_1, \gamma_2)_{max}$	S_1	S_2	γ_1	γ_2
51.1	0.088	0.24	52.348	51.592
114.5	0	0	100	100

The first row represents the values of parameters in the model to get minimum indemnity. Under this optimization structure, the maximum value of indemnity is 114.5 at the boundary values of parameters. Certainly, S_1 and S_2 getting value different than zero implies a lower indemnity. Optimization results for the remaining farms are summarized in Table 3.

Table 3: Optimization results for the TIGEM Farms

Local farm	$I(S_1, S_2, \gamma_1, \gamma_2)_{min}$ $I(S_1, S_2, \gamma_1, \gamma_2)_{max}$	S_1	S_2	γ_1	γ_2
Gökhöyük	79.9	-0.504	-0.504	100.005	100.003
	79.9	-0.504	-0.504	100.005	100.003
Ceylanpınar	83.93	-0.114	-0.114	100.002	100.002
	83.93	-0.114	-0.114	100.001	100.001
Bala	59.14	0.018	0.023	54.253	80.749
	86.5	0	0	100	100
Altınova	62.5	0.029	0.022	55.133	79.796
	90	0	0	100	100
Malya	72.94	0.069	0.022	86.431	93.453
	84.5	0	0	100	100
Gözlü	58.76	0.046	0.039	55.128	75.286
	90	0	0	100	100
Konuklar	106.58	-0.166	-0.166	100.001	100.002
	106.58	-0.166	-0.166	100.001	100.002
Kocaş	109.94	0.032	0.033	60.622	73.3
	161	0	0	100	100
Polatlı	72.59	0.042	0.027	83.118	90.151
	86.5	0	0	100	100
Ulaş	61.22	0.011	0.011	92.845	92.845
	67	0	0	100	100

The optimized $I(S_1, S_2, \gamma_1, \gamma_2)$ values are not plausible for the local farms Ceylanpınar, Konuklar and Gökhöyük. The reason of it can be the predetermined constraints. The constraints should be changed to derive meaningful results for these local farms too. Moreover, the first order optimality, the measure of how close any point to its optimal value, is not close to the value zero. It indicates some troubles about the optimization procedure.

Based on the values of $I(S_1, S_2, \gamma_1, \gamma_2)_{min}$ and $I(S_1, S_2, \gamma_1, \gamma_2)_{max}$, we derive a lower and an upper bound for the BRRP defined in (6). For the ELOP computation, the wheat price (WP) belonging to year 2005 is used for the design of insurance policy for the year 2006. The average price of all wheat types of provinces in 2005 is used for the selected local farms [5]. Moreover, forecasted wheat yield values, obtained for the year 2006 by using the best Box-Jenkins model, are summarized in Table 4. Based on these results, we estimate expected value of ELOP to calculate valid BRRP bounds for all farms. The nonzero solutions are presented in Table 5.

Table 4: Box-Jenkins summary

Local farm	TS process	FY (ton/ha)	OY (ton/ha)
Anadolu	ARIMA(1,1,1)	3.52	2.58
Gökhöyük	AR(1)	3.26	4.35
Ceylanpınar	IMA(1,1)	2.74	3.43
Bala	AR(1)	1.96	1.50
Altınova	AR(1)	2.20	2.91
Malya	IMA(1,1)	2.26	2.32
Gözlü	ARMA(1,1)	1.65	2.70
Konuklar	IMA(1,1)	2.12	4.22
Kocaş	ARI(1,1)	3.99	3.77
Polatlı	ARMA(1,1)	1.98	2.86
Ulaş	IMA(1,1)	2.19	3.42

TS : Time Series
FY : Forecasted Yield for the year 2006
OY : Observed Yield for the year 2006

Table 5: Valid BRRP bounds for farms

Station name	Lower bound	Upper bound
Anadolu	0.542	1.214
Bala	0.367	0.537
Kocaş	1.424	2.091

For some local farms, the BRRP ratio seems to be undefined bacause of the definition of ELOP value. Actually, we do not expect yield loss based on the difference between the FWY and OWY in 2006. For this reason, only nonzero and valid BRRP values are estimated for Anadolu, Bala and Kocaş Farms and given in

Table 5. The boundries for BRRP show that, the designed insurance policy is not much successful to cover the yield loss as we expected. Equivalently, the optimized $I(S_1, S_2, \gamma_1, \gamma_2)$ values are not efficient to compensate the expected yield loss under this model. Generally, it informs us on designing a more feasible index-based insurance for Turkey.

5 Conclusions

This paper implements Bayesian estimation technique to estimate the wheat yield in the design of index-based insurance in Turkey. A group of selected farms is studied for the application of the methodology. The pure premium and indemnity values are calculated. To determine the lower and upper bounds of the indemnity, a nonlinear optimization problem is employed under the assumptions on constraints.

The results have shown that the choice of the constraints has an important effect on plausible indemnity levels. For these reasons, the more sophisticated optimization models will be required to get more rigorous results. The sensitivity of indemnity under different constraint setups will be considered as a further research.

This study reveals also the efficiency and applicability of index-based insurance under drought risk in Turkey. Being at the stage of introducing drought insurance, assessment of basis risk under drought hazard requires more sophisticated modeling and better data-base planning.

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