An example

i	d_i	b_i	K_i	$\frac{d_i}{K_i}$
1	0.45	15	5	0.090
2	0.95	15	7.5	0.127
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$a = 180$, compute x_S^* for $S = \{1, 2, 4\}$.

- Take $T = \{4\}$ and compute

$$x_T = \sqrt{\frac{\sum_{i \in T} b_i d_i}{2a + \sum_{i \in T} b_i \frac{K_i^2}{d_i}}} = \sqrt{\frac{b_4 d_4}{2a + b_4 \frac{K_4^2}{d_4}}} = 0.11094.$$

Note that $S_{x_T} = \{i \in S \ : \ x_T < \frac{d_i}{K_i}\} = \{2, 4\}$ and $S_{x_T} \neq T$.

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For every ESL system (N, a, b, d, K) we can associate a cost game $(N, \hat{c})$ given by $\hat{c}(S) = C^S(x_S^*)$ for every non-empty $S \subset N$.

Theorem

Let (N, a, b, d, K) be an ESL system with associated cost game $\hat{c}$. Then, $\hat{c}$ is a concave game.

For every disjoint $S, T \subset N$ it holds that $\hat{c}(S \cup T) \leq \hat{c}(S) + \hat{c}(T)$.

$$Sh_i(\hat{c}) \in Core(\hat{c}) = \{x \in \mathbb{R}^N \mid \sum_{i \in N} x_i = \hat{c}(N), \sum_{i \in S} x_i \leq \hat{c}(S), \text{ for all } S\}.$$

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Another allocation in the core

For every ESL system (N, a, b, d, K),

$$\hat{c}(N) = \sum_{i \in i(N)} b_i \frac{d_i}{x_N^*} - \sum_{i \in i(N)} b_i K_i$$

among the agents in N.
The rule R assigns to every (N, a, b, d, K) with associated cost game $\hat{c}$ the allocation vector $R(\hat{c}) \in \mathbb{R}^n$ given by:

$$R_i(c) = \begin{cases} b_i[\frac{d_i}{x_N^*} - K_i] & \text{if } i \in i(N) \\ 0 & \text{otherwise.} \end{cases}$$

Theorem

Let (N, a, b, d, K) be an ESL system with associated cost game $\hat{c}$. Then $R(\hat{c}) \in Core(\hat{c})$.

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Let (N, a, b, d, K) be an ESL system with associated cost game $\hat{c}$. Then $R(\hat{c}) \in Core(\hat{c})$.

The example revisited

$a = 180$,

i	d_i	b_i	K_i	$\frac{d_i}{K_i}$
1	0.45	15	5	0.090
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4	1.20	12	9	0.133

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{4\}$	$\{1,2\}$	$\{1,3\}$	$\{1,4\}$	$\{2,3\}$
$C^S(x_S^*)$	14.750	20.865	20.896	21.800	20.865	20.896	21.800	21.924

S	$\{2,4\}$	$\{3,4\}$	$\{1,2,3\}$	$\{1,2,4\}$	$\{1,3,4\}$	$\{2,3,4\}$	N
$C^S(x_S^*)$	22.330	22.500	21.924	22.330	22.500	22.671	22.671

$$Sh(\hat{c}) = (3.69, 6.04, 6.14, 6.80)$$

$$R(\hat{c}) = (0, 5.13, 6.67, 10.87)$$

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SOME INVENTORY MODELS

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