Agent How much does it cost to place an order?

How much does it cost to store an item in a month?

How many items will I sell in a month?

Supplier

Demand

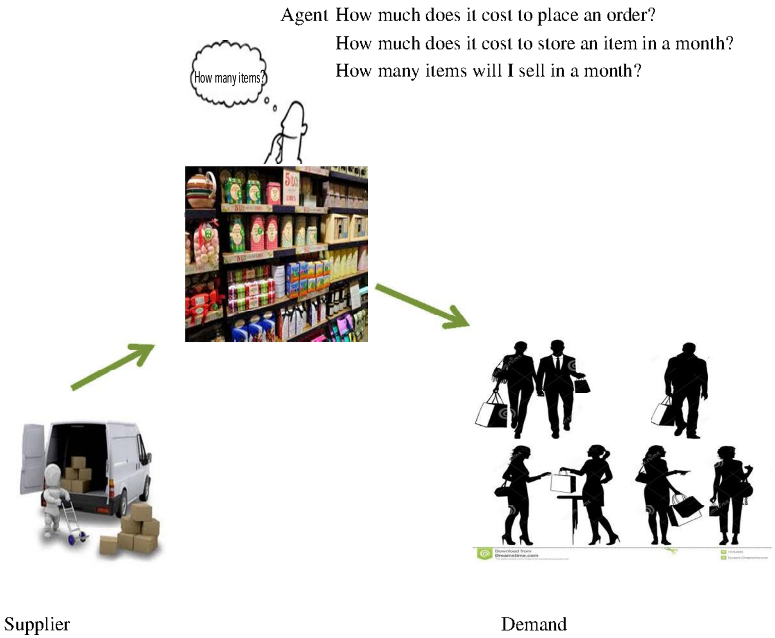
Agent How much does it cost to place an order?

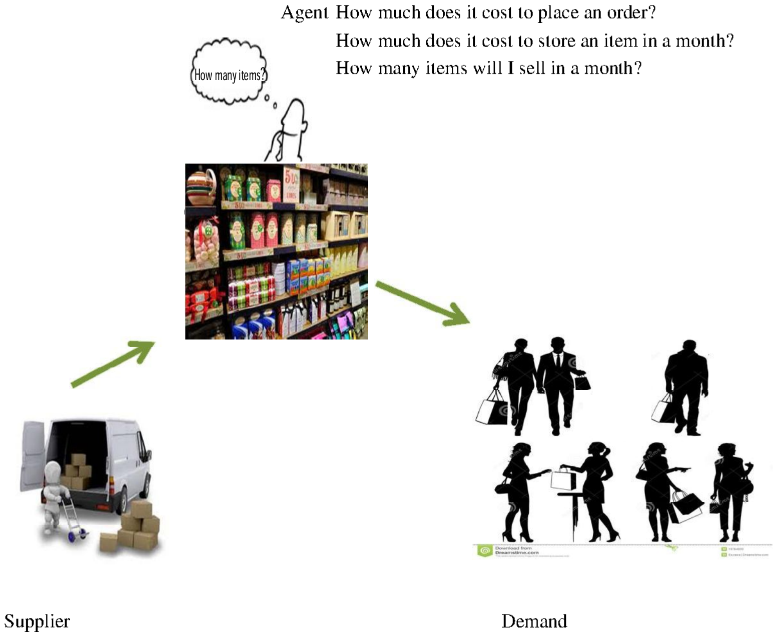
How much does it cost to store an item in a month?

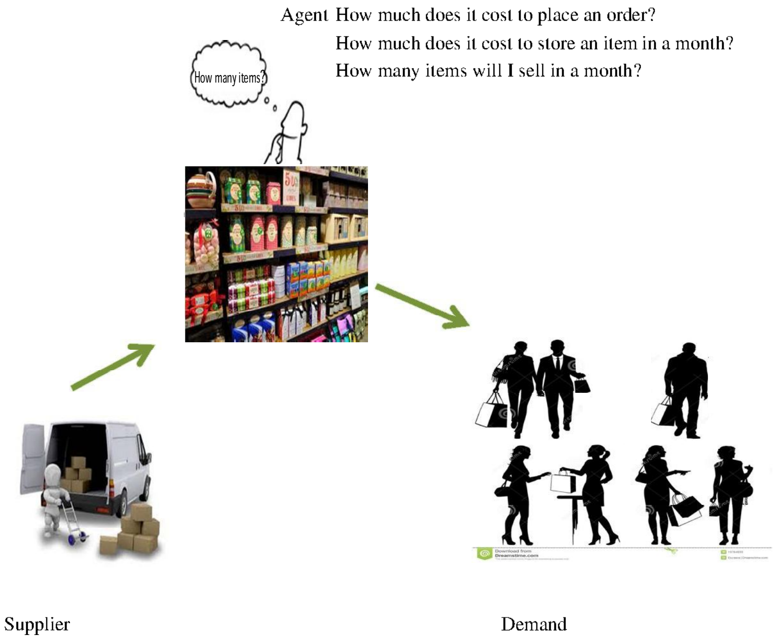
How many items will I sell in a month?

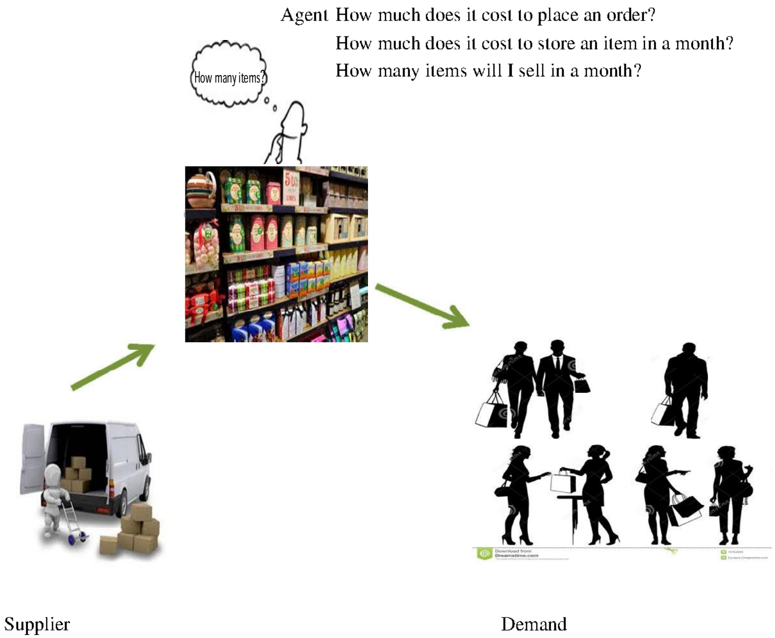
Supplier

Demand


Agent How much does it cost to place an order?
How much does it cost to store an item in a month?
How many items will I sell in a month?
How many items?
Supplier
Demand


Agent How much does it cost to place an order?
How much does it cost to store an item in a month?
How many items will I sell in a month?
How many items?
Supplier
Demand


Agent How much does it cost to place an order?
How much does it cost to store an item in a month?
How many items will I sell in a month?
How many items?
Supplier
Demand


Agent How much does it cost to place an order?
How much does it cost to store an item in a month?
How many items will I sell in a month?
How many items?
Supplier
Demand

A basic Economic Order Quantity problem (abbreviated basic EOQ problem) is a deterministic continuous review inventory problem with no shortages. The relevant parameters associated to one of these problems are:

- $a > 0$, the fixed cost per order,
- $h > 0$, the holding cost per item and per time unit,
- $d > 0$, the deterministic demand per time unit.

The objective is to determine the size of the order which allows to meet the demand and minimizes the average cost per time unit.

Remarks:

- An order is placed when the warehouse is empty.
- The lead time is constant (we assume zero).
- The warehouse has infinite capacity.

cycle: the length of the time interval between two consecutive orders.

A basic Economic Order Quantity problem (abbreviated basic EOQ problem) is a deterministic continuous review inventory problem with no shortages. The relevant parameters associated to one of these problems are:

- $a > 0$, the fixed cost per order,
- $h > 0$, the holding cost per item and per time unit,
- $d > 0$, the deterministic demand per time unit.

The objective is to determine the size of the order which allows to meet the demand and minimizes the average cost per time unit.

Remarks:

- An order is placed when the warehouse is empty.
- The lead time is constant (we assume zero).
- The warehouse has infinite capacity.

cycle: the length of the time interval between two consecutive orders.

A basic Economic Order Quantity problem (abbreviated basic EOQ problem) is a deterministic continuous review inventory problem with no shortages. The relevant parameters associated to one of these problems are:

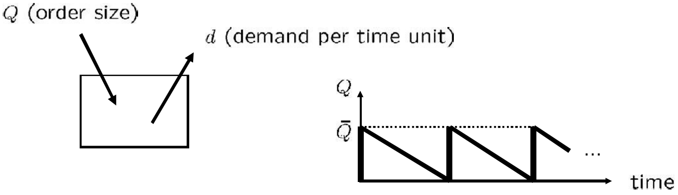
- $a > 0$, the fixed cost per order,
- $h > 0$, the holding cost per item and per time unit,
- $d > 0$, the deterministic demand per time unit.

The objective is to determine the size of the order which allows to meet the demand and minimizes the average cost per time unit.

Remarks:

- An order is placed when the warehouse is empty.
- The lead time is constant (we assume zero).
- The warehouse has infinite capacity.

cycle: the length of the time interval between two consecutive orders.



The optimal policy

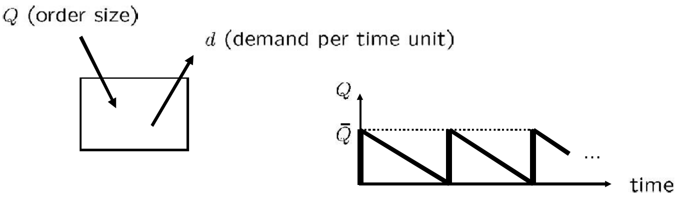
- $a > 0$, the fixed cost per order,
- $h > 0$, the holding cost per item and per time unit,
- $d > 0$, the deterministic demand per time unit,
- $x = d/Q$, the number of orders per time unit,
- $1/x = Q/d$, the cycle length,
- $Q/2$, the average inventory size in a cycle.

coste per cycle: $c(Q) = a + h\frac{Q}{2}\frac{Q}{d}$

coste per time unit:
$C(Q) = \frac{ad}{Q} + h\frac{Q}{2}$

Goal: $min\{C(Q) : Q > 0\}$

$$Q = \sqrt{\frac{2ad}{h}}, \quad x = \sqrt{\frac{dh}{2a}}, \quad C(Q) = 2ax$$



The optimal policy

- $a > 0$, the fixed cost per order,
- $h > 0$, the holding cost per item and per time unit,
- $d > 0$, the deterministic demand per time unit,
- $x = d/Q$, the number of orders per time unit,
- $1/x = Q/d$, the cycle length,
- $Q/2$, the average inventory size in a cycle.

coste per cycle: $c(Q) = a + h\frac{Q}{2}\frac{Q}{d}$

coste per time unit:
$C(Q) = \frac{ad}{Q} + h\frac{Q}{2}$

Goal: $min\{C(Q) : Q > 0\}$

$$Q = \sqrt{\frac{2ad}{h}}, \quad x = \sqrt{\frac{dh}{2a}}, \quad C(Q) = 2ax$$

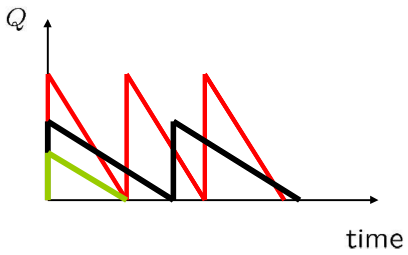
Basic EOQ systems

A basic EOQ system is a multiple agent situation where each agent faces a basic EOQ problem.

N denotes the finite set of agents.

The parameters associated to every $i \in N$ in one of these systems are:

- $a > 0$, the fixed cost per order,
- $h_i > 0$, the holding cost per item and per time unit,
- $d_i > 0$, the deterministic demand per time unit.



Can they save costs by placing joint orders?

Agents in a basic EOQ systems cooperate

Cooperation: they place joint orders.

Take $i \in N$. Then, $\frac{Q_j}{d_j} = \frac{Q_i}{d_i}$, for all $j \in N$.

- $C(Q_i) = \frac{ad_i}{Q_i} + \sum_{j \in N} h_j \frac{Q_j}{2} = \frac{ad_i}{Q_i} + \frac{Q_i}{2d_i} \sum_{j \in N} d_j h_j$.
- $Q_i^* = \sqrt{\frac{2ad_i^2}{\sum_{j \in N} d_j h_j}}$.
- $Q_k^* = \sqrt{\frac{2ad_i^2}{\sum_{j \in N} d_j h_j}} \frac{d_k}{d_i} = \sqrt{\frac{2ad_k^2}{\sum_{j \in N} d_j h_j}}$, for all $k \in N$.
- $\bar{x}_k^* = \sqrt{\frac{\sum_{j \in N} d_j h_j}{2a}}$, for all $k \in N$.
- $C(Q_i^*) = 2a\sqrt{\sum_{j \in N} \bar{x}_j^2}$.

Do they really save costs?

If so, how do they share the costs?

An EOQ model

Let (N, a, h, d) be a basic EOQ system, where h and d denote the vectors $(h_i(_{i\in N}$ and $(d_i)_{i\in N}$, respectively. Notice that, for every $S \subset N$ and every $i \in S$,

- $c(i) := C(\bar{Q}_i) = 2a\bar{x}_i$.
- $c(S) := C(Q_i^*) = 2a\sqrt{\sum_{j\in S}\bar{x}_i^2} = \sqrt{\sum_{i\in S} c(i(^2}$.

Definition

An inventory game is a cost game c of the family

$$\{c \in G^N \subset \mathcal{G} \mid c(S(^2 = \sum_{i\in S} c(i)^2 \text{ and } c(S) > 0,\ \forall\, \emptyset \neq S \subset N\}.$$

Theorem

Every inventory game is concave.

Do they really save costs?

$c(S) + c(T(\leq c(S \cup T($, for all $S, T \subset N$, $S \cap T = \emptyset$.

YES, they do!!

How do they share the total cost?

- The Shapley value.
- Any allocation in the Core.

  ⋮

A nice allocation: The SOC rule

(N, a, h, d) an EOQ system and (N, c) its associated inventory game. Take $i \in N$. Agent i pays

- his own holding cost.
- a part of the fixed ordering cost proportional to its parameter $\bar{x}_i^2$

$$SOC_i(N, a, h, d) = a \frac{d_i}{Q_i^*} \frac{\bar{x}_i^2}{\sum_{j \in N} \bar{x}_j^2} + h_i \frac{Q_i^*}{2}$$

$$SOC_i(N, a, h, d) = \frac{c(i)^2}{\sum_{j \in N} c(j)^2} c(N).$$

It always belongs to the core.