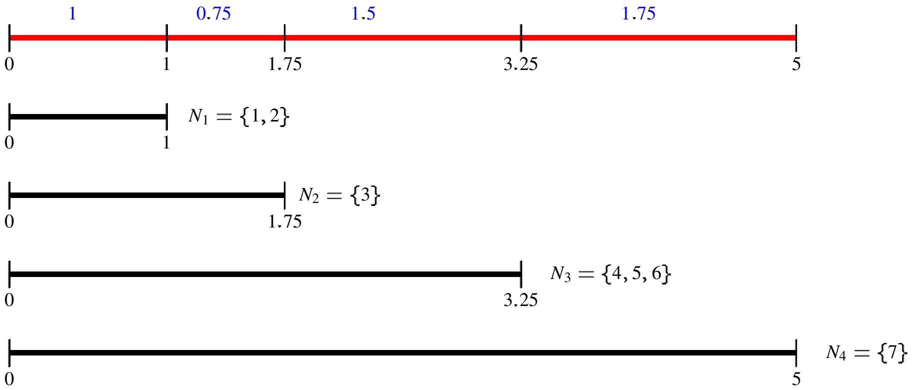
A classical example: Airport game



The game.

$N = \{1, 2, 3, 4, 5, 6, 7\} = \cup_{k \in K} N_k$, $K = \{1, 2, 3, 4\}$
$c_1 = c_2 = 1$, $c_3 = 1.75$, $c_4 = c_5 = c_6 = 3.25$, $c_7 = 5$;

For every $S \subset N$,

$$c(S) = \max\{c_k \ : \ N_k \cap S \neq \emptyset\}.$$

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$c(1,2) = 1$, $c(1,3) = 1.75$, $c(1,4) = 3.25$, and so on

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(N, c) an aiport game.

- (N, c) is subadditive.
- (N, c) is concave.
- $Core(N, c)$ is nonempty.
- The Shapley value belongs to the core. For each $i \in N$,

$$\Phi_i(N, c) = \sum_{k=1}^{k(i)} \frac{c_k - c_{k-1}}{n_k}$$

being $K(i) \in K$ such that $i \in N_{k(i)}$ and $n_k = | \cup_{l=k}^{|K|} N_l |$

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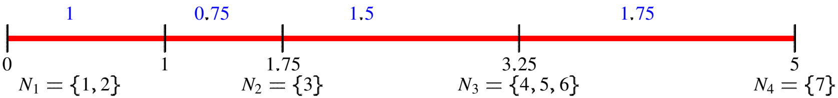

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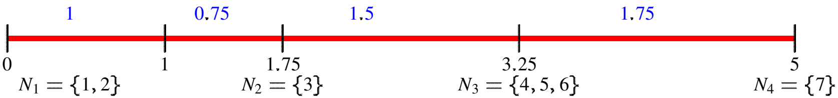

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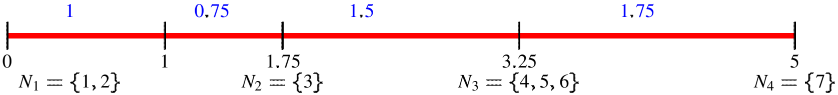

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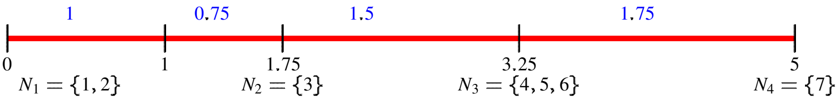

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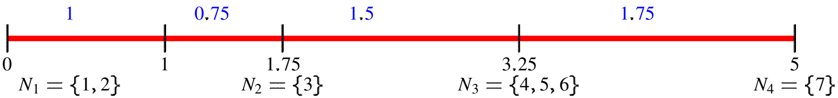

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A Linear Programming Game

A set of suppliers: I. A set of demand points: J.

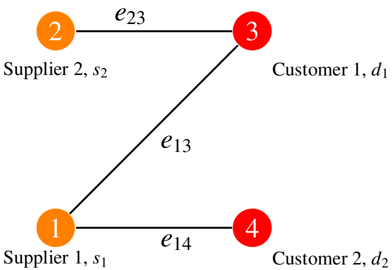
For each $i \in I$:

- s_i is the supply volume.
- J_i is the set of demand points that can be reached from supply point i.

For each $j \in J$:

- d_j is the demand volume.
- I_j is the set of supply points that can be used to satisfy the demand point j.

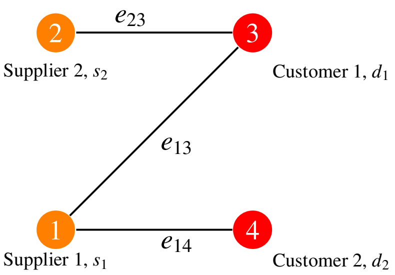
For each $i \in I$ and $j \in J$, e_{ij} is the unit cost of flow between i and j.



$I = \{\text{Supplier 1, Supplier 2}\} = \{1,2\}$, $J = \{\text{Customer 1, Customer 2}\} = \{3,4\}$

$I_3 = \{1,2\}$, $I_4 = \{1\}$, $J_1 = \{3,4\}$, $J_2 = \{3\}$,

A Linear Programming Game



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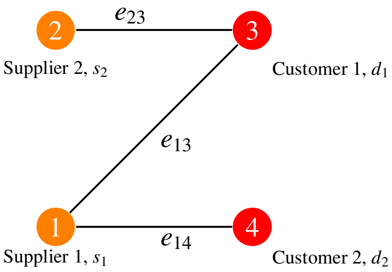
$I_3 = \{1, 2\}$, $I_4 = \{1\}$, $J_1 = \{3, 4\}$, $J_2 = \{3\}$,

The problem

$$
\begin{array}{lll}
\min & \sum_{i \in I} \sum_{j \in J_i} e_{ij} w_{ij}, & \\
s.t.: & \sum_{j \in J_i} w_{ij} \leq s_i, & \text{for all } i \in I, \\
 & \sum_{i \in I_j} w_{ij} = d_j, & \text{for all } j \in J, \\
 & w_{ij} \geq 0 & \text{for all } i \in I,\ j \in J_i.
\end{array}
$$

A Linear Programming Game



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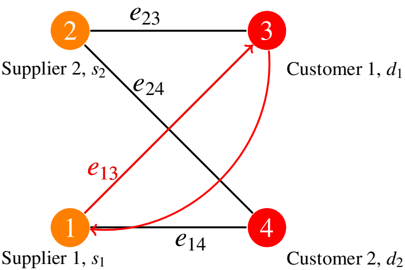

$\mathbf{s_1 - d_1 - s_1}$

$\mathbf{s_1 - d_1 - s_2 - d_2 - s_1}$

The problem

$$
\begin{aligned}
\min \quad & \sum_{r\in D} c_{rd}x_{rd} + \sum_{k\in B} c_{kb}x_{kb}, & \\
s.t.: \quad & & \\
& \sum_{r\in D} a^d_{ir}x_{rd} + \sum_{k\in B} a^b_{ik}x_{kb} \le s_i, & \text{for all } i \in I, \\
& \sum_{r\in D} b^d_{jr}x_{rd} + \sum_{k\in B} b^b_{jk}x_{kb} \ge d_j, & \text{for all } j \in J, \\
& x_{rd} \ge 0 & \text{for all } r \in D, \\
& x_{rb} \ge 0 & \text{for all } r \in B.
\end{aligned}
$$

$$
a^d_{ir} = \begin{cases} 1 \text{ if direct trip picks up at } i, \\ 0 \text{ otherwise} \end{cases}
$$

$$
a^b_{ir} = \begin{cases} 1 \text{ if backhaul route picks up at } i, \\ 0 \text{ otherwise} \end{cases}
$$

$$
b^d_{jr} = \begin{cases} 1 \text{ if direct trip delivers at } j, \\ 0 \text{ otherwise} \end{cases}
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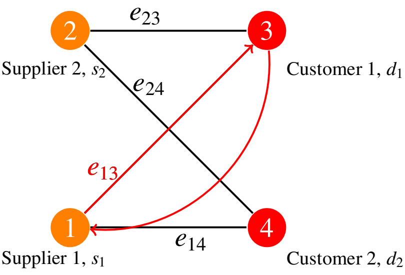
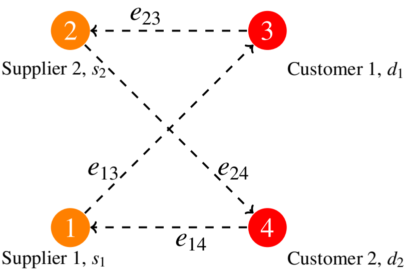
A Linear Programming Game



$\mathbf{s_1 - d_1 - s_1}$



$\mathbf{s_1 - d_1 - s_2 - d_2 - s_1}$

The problem

$$
\begin{array}{lll}
\min & \sum_{r\in D} c_{rd}x_{rd} + \sum_{r\in B} c_{kb}x_{kb}, & \\
s.t.: & & \\
& \sum_{r\in D} a^d_{ir}x_{rd} + \sum_{r\in B} a^b_{ik}x_{kb} \le s_i, & \text{for all } i \in I, \\
& \sum_{r\in D} b^d_{jr}x_{rd} + \sum_{r\in B} b^b_{jk}x_{kb} \ge d_j, & \text{for all } j \in J, \\
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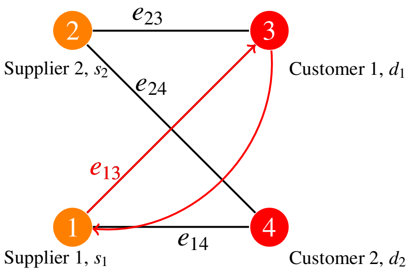
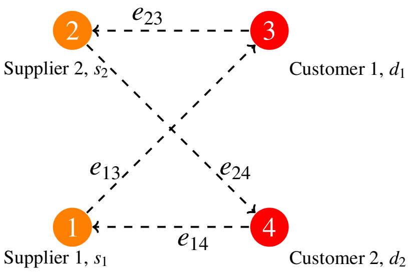


$\mathbf{s_1 - d_1 - s_1}$



$\mathbf{s_1 - d_1 - s_2 - d_2 - s_1}$

The problem

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i\in D} c_{rd}x_{rd} + \sum_{k\in B} c_{kb}x_{kb}, & \\
s.t.: \quad & & \\
& \textstyle\sum_{r\in D} a^d_{ir}x_{rd} + \sum_{k\in B} a^b_{ik}x_{kb} \leq s_i, & \text{for all } i \in I, \\
& \textstyle\sum_{r\in D} b^d_{jr}x_{rd} + \sum_{k\in B} b^b_{jk}x_{kb} \geq d_j, & \text{for all } j \in J, \\
& x_{rd} \geq 0 & \text{for all } r \in D, \\
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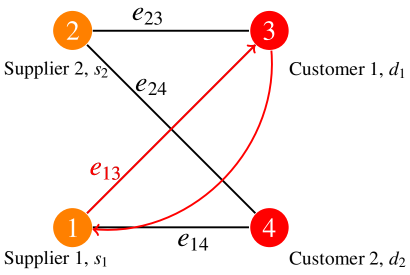
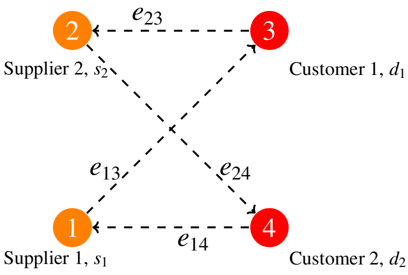


$\mathbf{s_1 - d_1 - s_1}$



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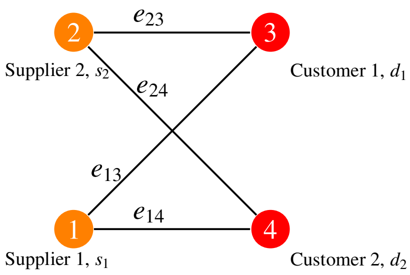
$$

A Linear Programming Game

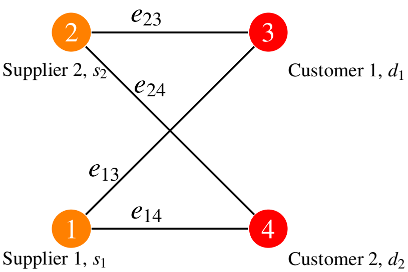


$N = \{s_1, s_2, d_1, d_2\}$,
Direct paths: $s_1 - d_1 - s_1$, $s_1 - d_2 - s_1$, $s_2 - d_1 - s_2$, $s_2 - d_2 - s_2$,
Backhaul paths: $s_1 - d_1 - s_2 - d_2 - s_1$, $s_1 - d_2 - s_2 - d_1 - s_1, \ldots$

$$
\begin{array}{lll}
\min & \sum_{i \in D} c_{rd} x_{rd} + \sum_{k \in B} c_{kb} x_{kb}, & \\
s.t.: & & \\
& \sum_{r \in D} a^d_{ir} x_{rd} + \sum_{k \in B} a^b_{ik} x_{kb} \leq s_i, & \text{for all } i \in I, \\
& \sum_{r \in D} b^d_{jr} x_{rd} + \sum_{k \in B} b^b_{jk} x_{kb} \geq d_j, & \text{for all } j \in J, \\
& x_{rd} \geq 0 & \text{for all } r \in D, \\
& x_{rb} \geq 0 & \text{for all } r \in B.
\end{array}
$$

$$
a^d_{ir} = \begin{cases} 1 \text{ if direct trip picks up at } i, \\ 0 \text{ otherwise} \end{cases}
\quad
a^b_{ir} = \begin{cases} 1 \text{ if backhaul route picks up at } i, \\ 0 \text{ otherwise} \end{cases}
$$

$$
b^d_{jr} = \begin{cases} 1 \text{ if direct trip delivers at } j, \\ 0 \text{ otherwise} \end{cases}
\quad
b^b_{jr} = \begin{cases} 1 \text{ if backhaul route delivers at } j, \\ 0 \text{ otherwise} \end{cases}
$$



$N = \{s_1, s_2, d_1, d_2\}$,
Direct paths: $s_1 - d_1 - s_1$, $s_1 - d_2 - s_1$, $s_2 - d_1 - s_2$, $s_2 - d_2 - s_2$,
Backhaul paths: $s_1 - d_1 - s_2 - d_2 - s_1$, $s_1 - d_2 - s_2 - d_1 - s_1, \ldots$

$$\begin{array}{lll}
\min & \sum_{i\in D} c_{rd}x_{rd} + \sum_{k\in B} c_{kb}x_{kb}, & \\
s.t.: & & \\
& \sum_{r\in D} a^d_{ir}x_{rd} + \sum_{k\in B} a^b_{ik}x_{kb} \leq s_i, & \text{for all } i \in I, \\
& \sum_{r\in D} b^d_{jr}x_{rd} + \sum_{k\in B} b^b_{jk}x_{kb} \geq d_j, & \text{for all } j \in J, \\
& x_{rd} \geq 0 & \text{for all } r \in D, \\
& x_{rb} \geq 0 & \text{for all } r \in B.
\end{array}$$

$$a^d_{ir} = \begin{cases} 1 \text{ if direct trip picks up at } i, \\ 0 \text{ otherwise} \end{cases}$$

$$a^b_{ir} = \begin{cases} 1 \text{ if backhaul route picks up at } i, \\ 0 \text{ otherwise} \end{cases}$$

$$b^d_{jr} = \begin{cases} 1 \text{ if direct trip delivers at } j, \\ 0 \text{ otherwise} \end{cases}$$

$$b^b_{jr} = \begin{cases} 1 \text{ if backhaul route delivers at } j, \\ 0 \text{ otherwise} \end{cases}$$

$$\begin{aligned}
\min \quad & \textstyle\sum_{i \in D} c_{rd} x_{rd} + \sum_{k \in B} c_{kb} x_{kb}, \\
s.t.: \quad & \\
& \textstyle\sum_{r \in D} a^d_{ir} x_{rd} + \sum_{k \in B} a^b_{ik} x_{kb} \le s_i, && \text{for all } i \in I, \\
& \textstyle\sum_{r \in D} b^d_{jr} x_{rd} + \sum_{k \in B} b^b_{jk} x_{kb} \ge d_j, && \text{for all } j \in J, \\
& x_{rd} \ge 0 && \text{for all } r \in D, \\
& x_{rb} \ge 0 && \text{for all } r \in B.
\end{aligned}$$

$$a^d_{ir} = \begin{cases} 1 \text{ if direct trip picks up at } i, \\ 0 \text{ otherwise} \end{cases}$$

$$a^b_{ir} = \begin{cases} 1 \text{ if backhaul route picks up at } i, \\ 0 \text{ otherwise} \end{cases}$$

$$b^d_{jr} = \begin{cases} 1 \text{ if direct trip delivers at } j, \\ 0 \text{ otherwise} \end{cases}$$

$$b^b_{jr} = \begin{cases} 1 \text{ if backhaul route delivers at } j, \\ 0 \text{ otherwise} \end{cases}$$

An allocation: Equal Profit Method

$$\begin{aligned}
\min \quad & f, \\
s.t.: \quad & \\
& f \ge \frac{y_i}{c(i)} - \frac{y_j}{c(j)}, && \text{for all } i, j \text{ with } i \neq j, \\
& \textstyle\sum_{i \in S} y_i \le c(S), && \text{for all } S \subset N, \\
& y_i \ge 0, && \text{for all } i \in N.
\end{aligned}$$

Bibliography

Frisk, M., Gothe-Lundgren, M., Jornsten, K., and Ronnqvist, M. (2010).
Cost allocation in collaborative forest transportation.
European Journal of Operational Research, 205(2):448 – 458.

González-Díaz, J., Fiestras-Janeiro, M., and García-Jurado, I. (2010).
An Introductory Course on Mathematical Game Theory, volume 115 of Graduate Studies in Mathematics.
American Mathematical Society.

Littlechild, S. C. and Owen, G. (1973).
A simple expression for the Shapley value in a special case.
Management Science, 20:370–372.

Myerson, R. (1980).
Conference structures and fair allocation rules.
International Journal of Game Theory, 9(3):169–182.

Myerson, R. B. (1991).
Game Theory: Analysis of Conflict.
Harvard University Press.

Osborne, M. and Rubisntein, A. (1994).
A Course in Game Theory.
MIT Press.

Shapley, L. S. (1953).
A value for n-person games.
In Kuhn, H. and Tucker, A., editors, Contributions to the theory of games II, volume 28 of Annals of Mathematics Studies. Princeton University Press, Princeton.

MODELLING COOPERATION

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SOME INVENTORY MODELS

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Outline

An EOQ model

Basic EOQ system without holding costs

Basic EOQ system without holding costs and with transportation costs

An Economic Shortage Level problem