EURO Summer Institute Lleida 2014

Grup de Computació Distribuïda

Parallelization of the CBD algorithm to speed up the optimization of supply chain models

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About Me

University of Lleida	ETIG in computer engineering degree 2006 - 2009
	B.S in computer engineering degree 2010 - 2012
	Master in computer engineering degree 2012 - 2013
	Coursing my PhD 2013

Linkedin

ResearchGate

Outline

1. Introduction

- Motivation
- Objectives

2. Theoretical Background

- Mathematical Background
- Benders Decomposition
- Cluster Benders Decomposition

3. Parallel Cluster Benders Decomposition

- Implementation
- Master
- Workers

Outline

4. Computational Results

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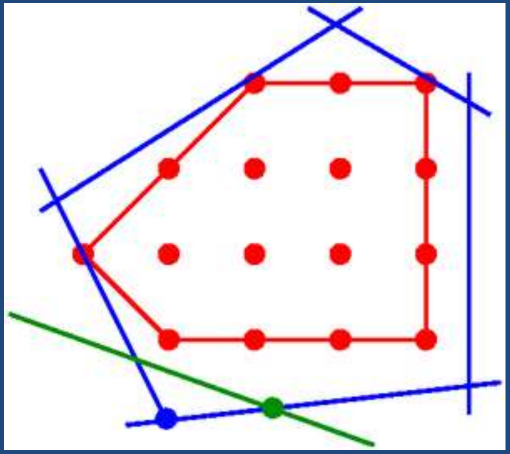
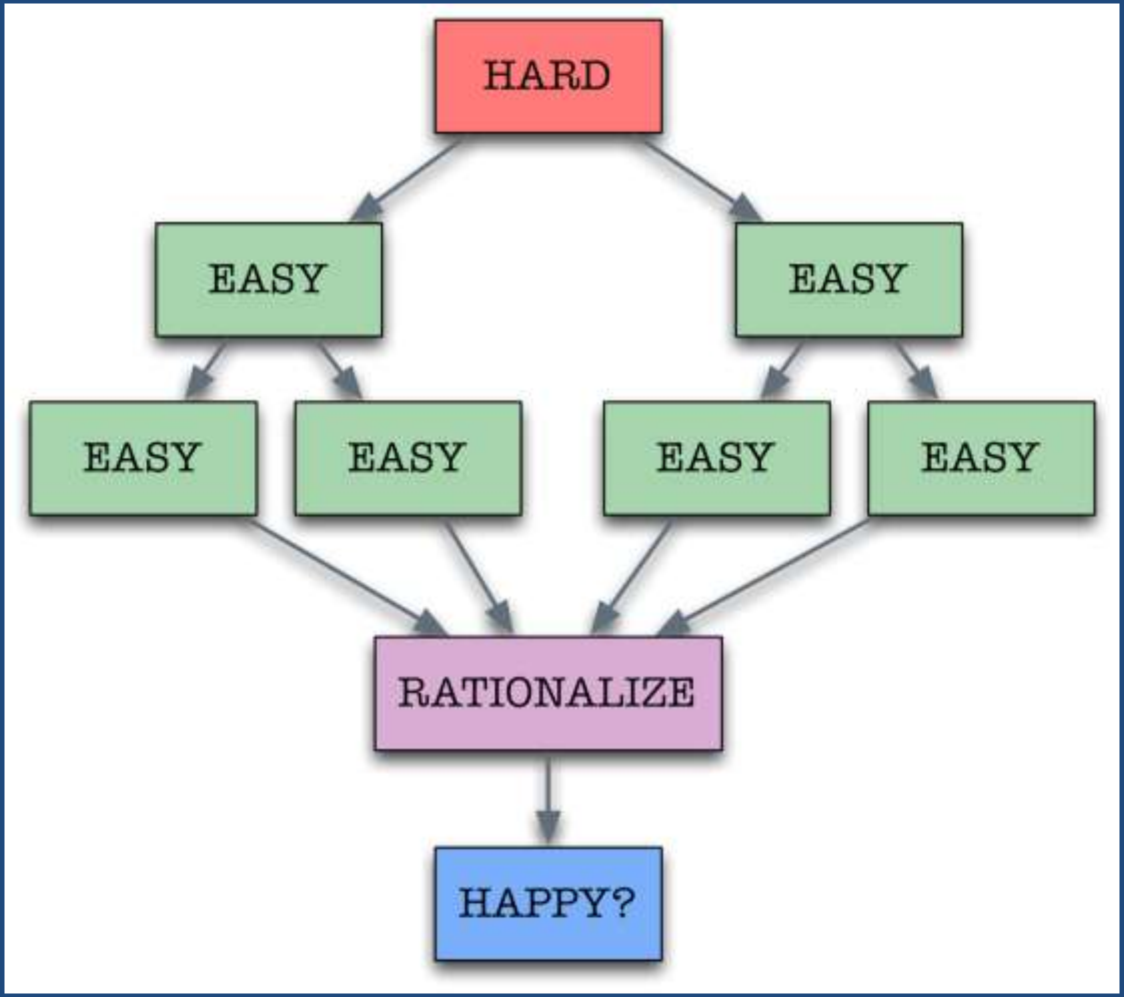
- Environment
- Small Instances
- Medium Instances

5. Future Research

- New type of problems.
- Esteve's farm models.

Motivation





"Whatever you do will be insignificant but it is very important that you do it."

Objectives

- Speed Up the resolution time of the CBD.
- Adapt the algorithm to fit well with real problems (e.g. Esteve's models).
- Develop a easy tool for users (e.g. to pig companies for optimal planning).

Stochastic LP

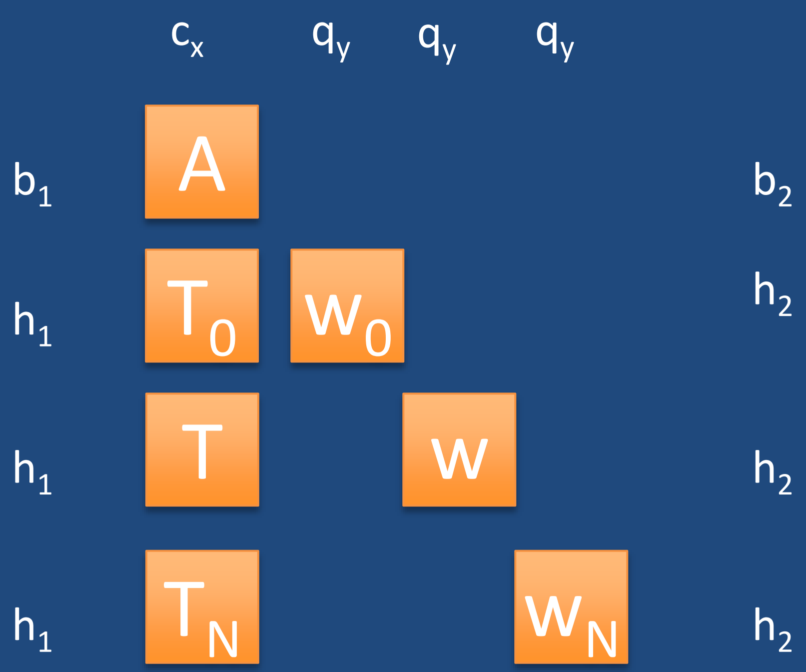
$$\min \quad cx + \sum_{k=1}^{K} p_k q_k^T y_k$$

$st:$

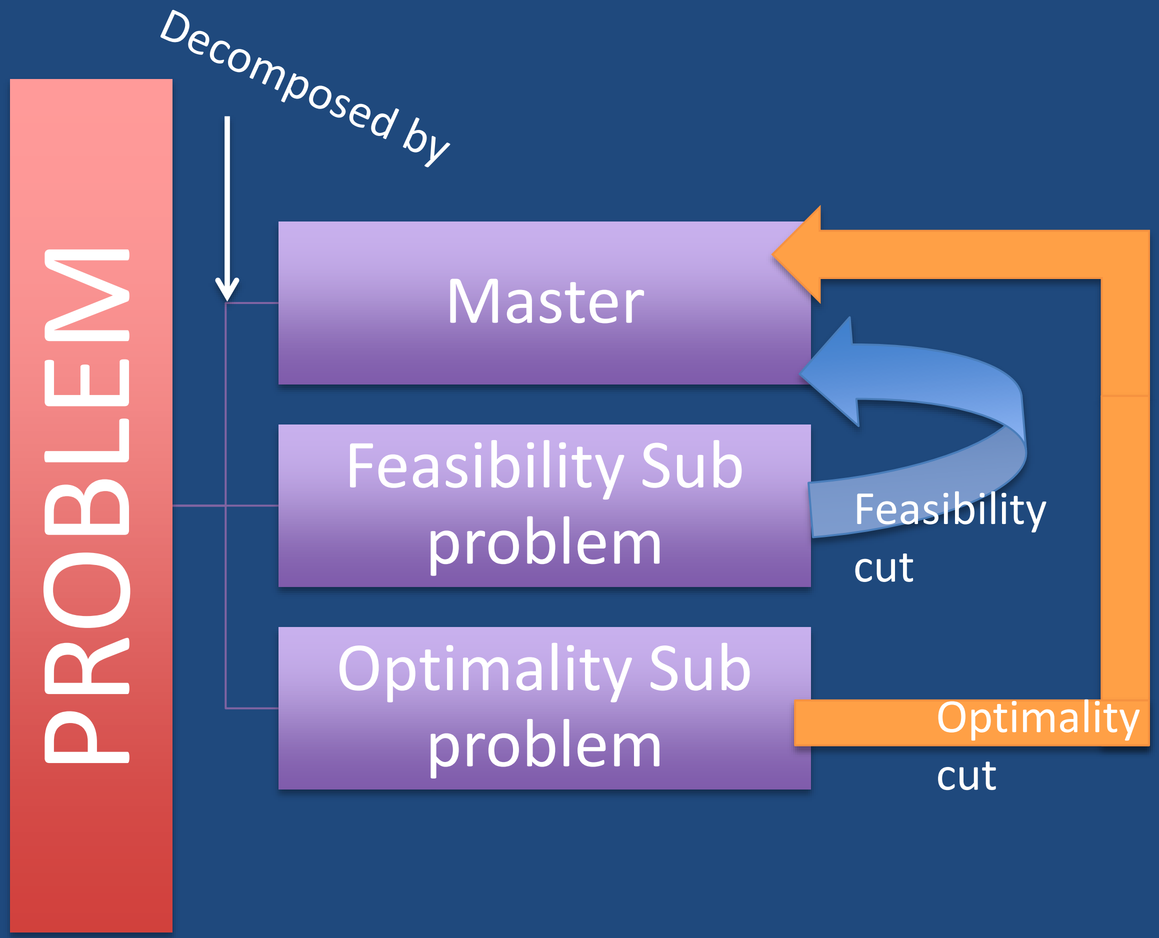
$$b_1 \leq A_x \leq b_2$$

$$h_k^1 \leq T_k x \leq h_k^2 \quad \forall k \in \Omega$$

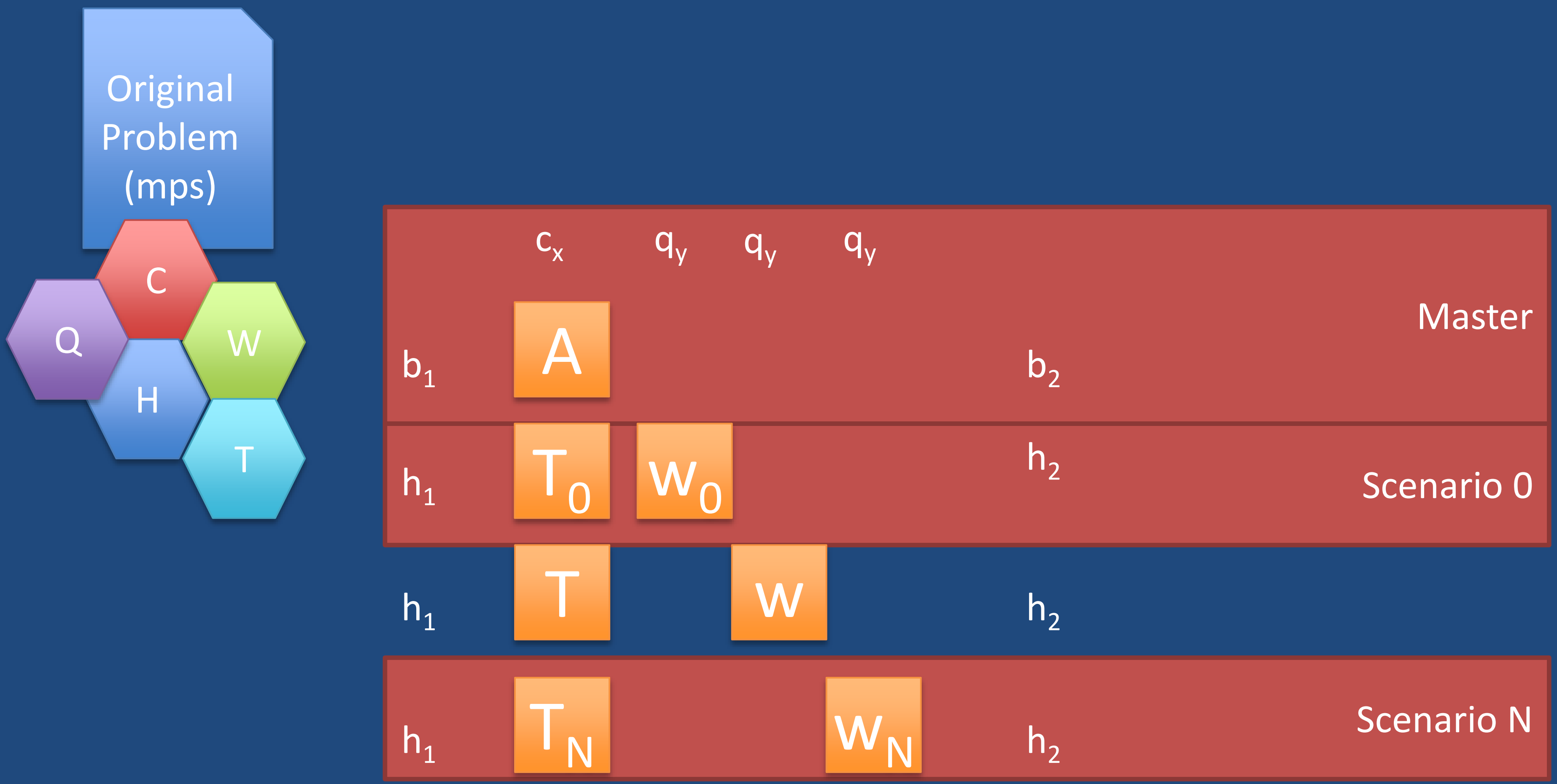
$$x, y_k \geq 0$$



Schema


Decomposed by
PROBLEM
Master
Feasibility Sub problem
Optimality Sub problem
Feasibility cut
Optimality cut

Schema


Original
Problem
(mps)
C
Q
W
H
T
cx
qy
qy
qy
Master
b1
A
b2
h1
T0
W0
h2
Scenario 0
h1
T
W
h2
h1
TN
WN
h2
Scenario N

Master Problem

Master

$$\min \quad z = c^T x + q$$

Objective Function

$$s.t.:$$

$$Ax = b$$

First Stage Constraints

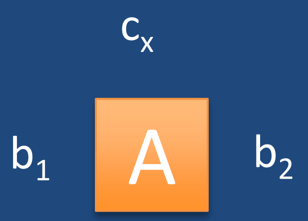
$$D_l x \geq d_l \quad l = 1,\ldots,r$$

Feasible Cut

$$E_l x + q \geq e_l \quad l = 1,\ldots,s$$

Optimality Cut

$$x \geq 0 \quad q \in \mathbb{R}$$



Feasibility Subproblem

FWorker

$$D_{r+1} = (s^v)^T T_k$$

$$d_{r+1} = (s^v)^T h_k$$

$$\min \quad w' = e^T v^+ + e^T v^-$$

$$s.t.:$$

$$Wy + Iv^+ - Iv^- = h_k - T_k x^v$$ Second Stage Constraints

$$y \geq 0 \quad v^+ \geq 0 \quad v^- \geq 0$$

$$e^T = (1, \ldots, 1)$$

$$D_l x \geq d_l \quad l = 1, \ldots, r$$ Ensure the feasibility of the MP solution

c_x q_y

h_1 T W h_2

Optimality Subproblem

OWorker

$$E_{s+1} = \sum_{k=1}^{K} p_k \cdot (\pi_k^v)^T T_k$$

$$e_{s+1} = \sum_{k=1}^{K} p_k \cdot (\pi_k^v)^T h_k$$

$$\min \quad w = q_k^T y$$

$$s.t$$

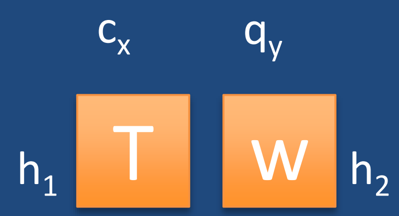
$$W_y = h_k - T_k x^v$$

Second Stage Constraints

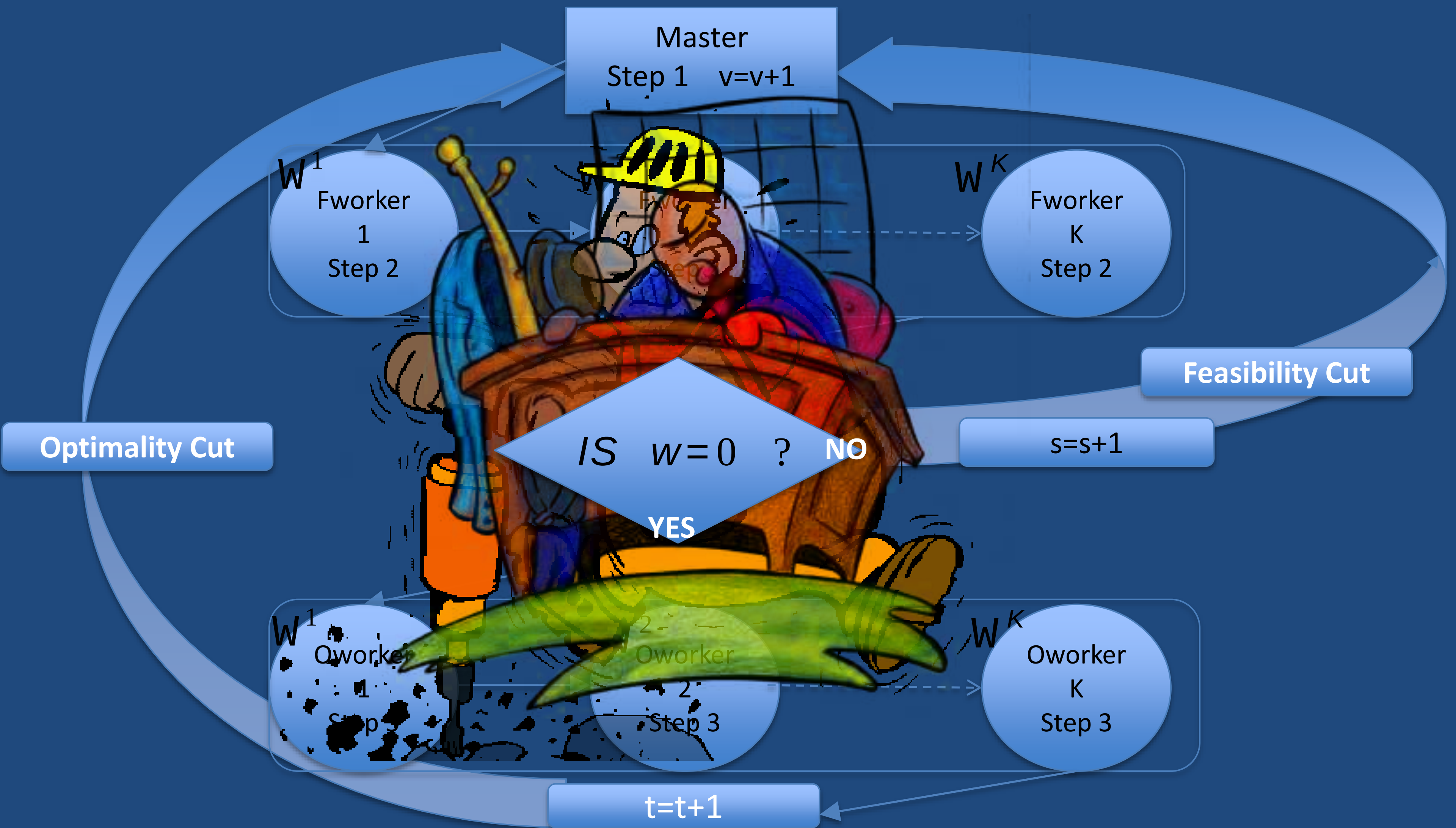
$$y \geq 0$$

$$E_l x + q \geq e_l \quad l = 1, \ldots, s$$

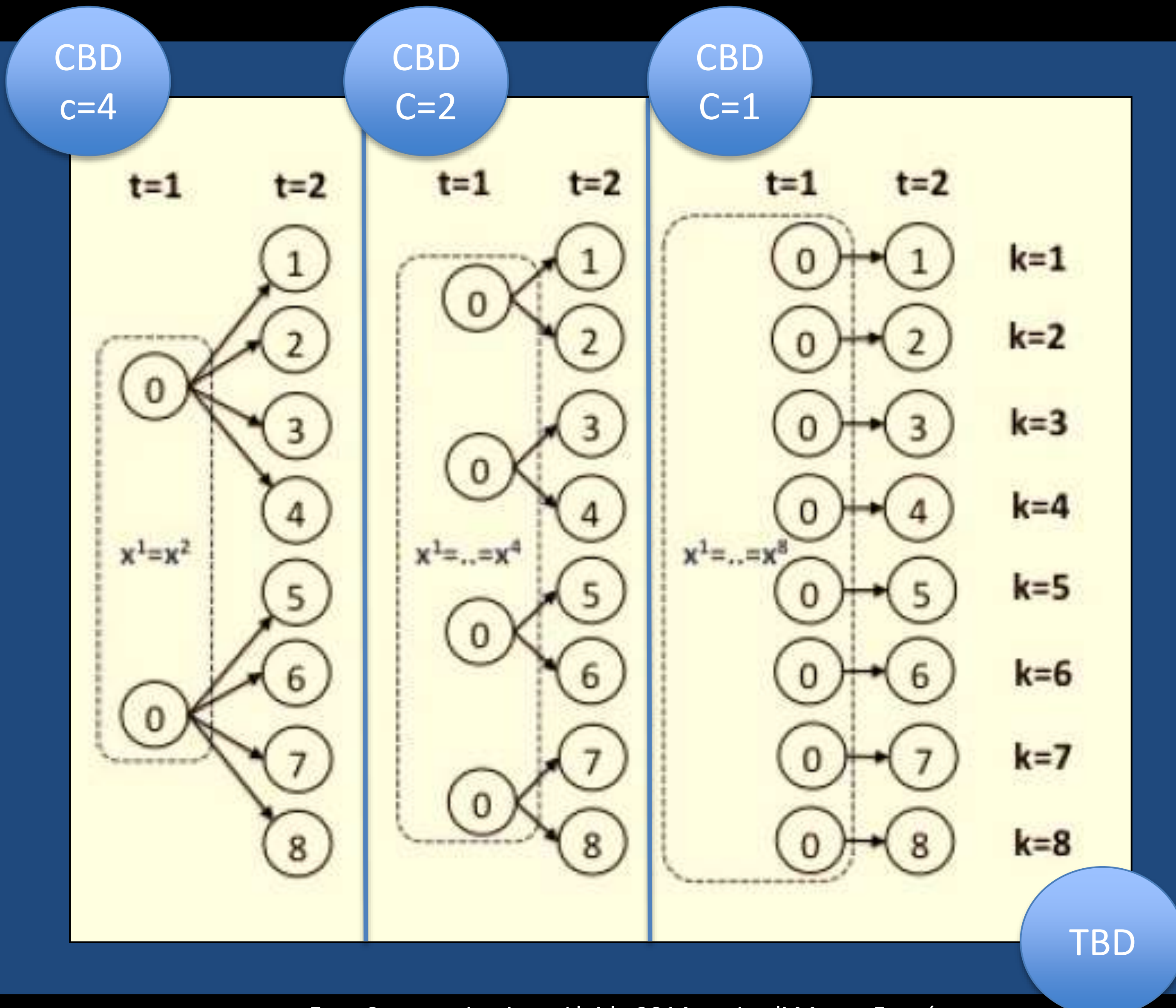
Represents a lower bound



Algorism


Master
Step 1 v=v+1
W^1
Fworker
1
Step 2
W^K
Fworker
K
Step 2
Feasibility Cut
Optimality Cut
IS $w=0$?
NO
s=s+1
YES
W^1
Oworker
1
W^K
Oworker
K
Step 3
Oworker
2
Step 3
t=t+1

Algorism CBD



Implementation

Classical Master-Workers

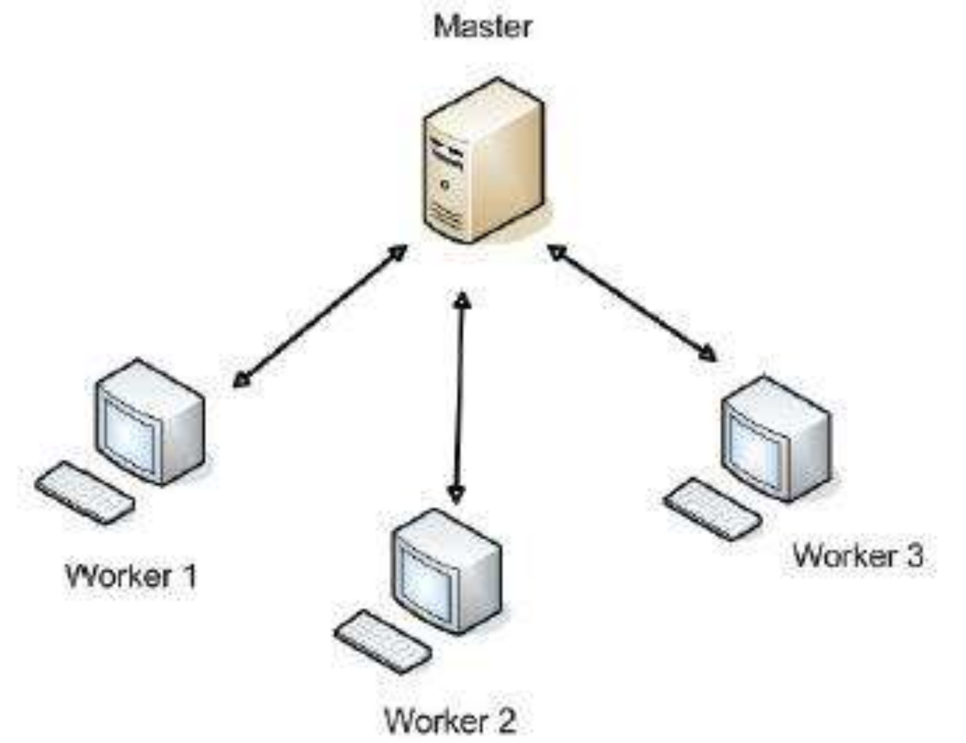


Figure 2: A Scheme for the Master Worker Architecture.

Pre-calculations

Use of templates

Avoid Memory Problems

Gain efficiency

Speed up the resolution

Algorithm PCBD

```
LB = 0, UB = ∞;
while LB ≤ UB − ε do
    STEP 1: Solve the reduced master problem (MPc);
    Update LB with the value θ^v;
    for all scenario cluster do
        Generate the worker thread (FWorker);
    end for
    Wait for the results of all the threads (FWorker) generated;
    if f.o_p ≠ 0 then
        Add the feasible cut (15) to the problem (MPc);
        Return to STEP 1 and solve the problem (MPc);
    else
        for all scenario do
            Generate the worker thread (OWorker);
        end for
        Wait for the results of all the threads (OWorker) generated;
        Update UB with the value w;
        if LB ≤ UB − ε then
            Add the optimality cut (16) to the problem (MPc);
        end if
        Return to STEP 1 and solve the problem (MPc);
    end if
end while
```

Algorithm PCBD

FWorkers

1: Receive the master information;
2: Generate the feasibility subproblem (FCUT);
3: Solve the feasibility subproblem (FCUT);
4: Send the solution to the master thread;

OWorkers

1: Receive the master information;
2: Generate the optimality subproblem (OCUT);
3: Solve the optimality subproblem (OCUT);
4: Send the solution to the master thread;

Codes

TBD – Traditional Benders Decomposition

- Serial
- Parallel (Threads, MPI)

CBD – Cluster Benders Decomposition

- Serial
- Parallel (Threads, MPI)

Environment

8 CPU'S

3,4 GHz

16 GB RAM

500 GB HDD

O.S.:	**Ubuntu 13.04**
Language:	**C++**
Solver:	**Cplex 12.6**
Architecture:	**64 bits**

Workstation Dell Precision T1600

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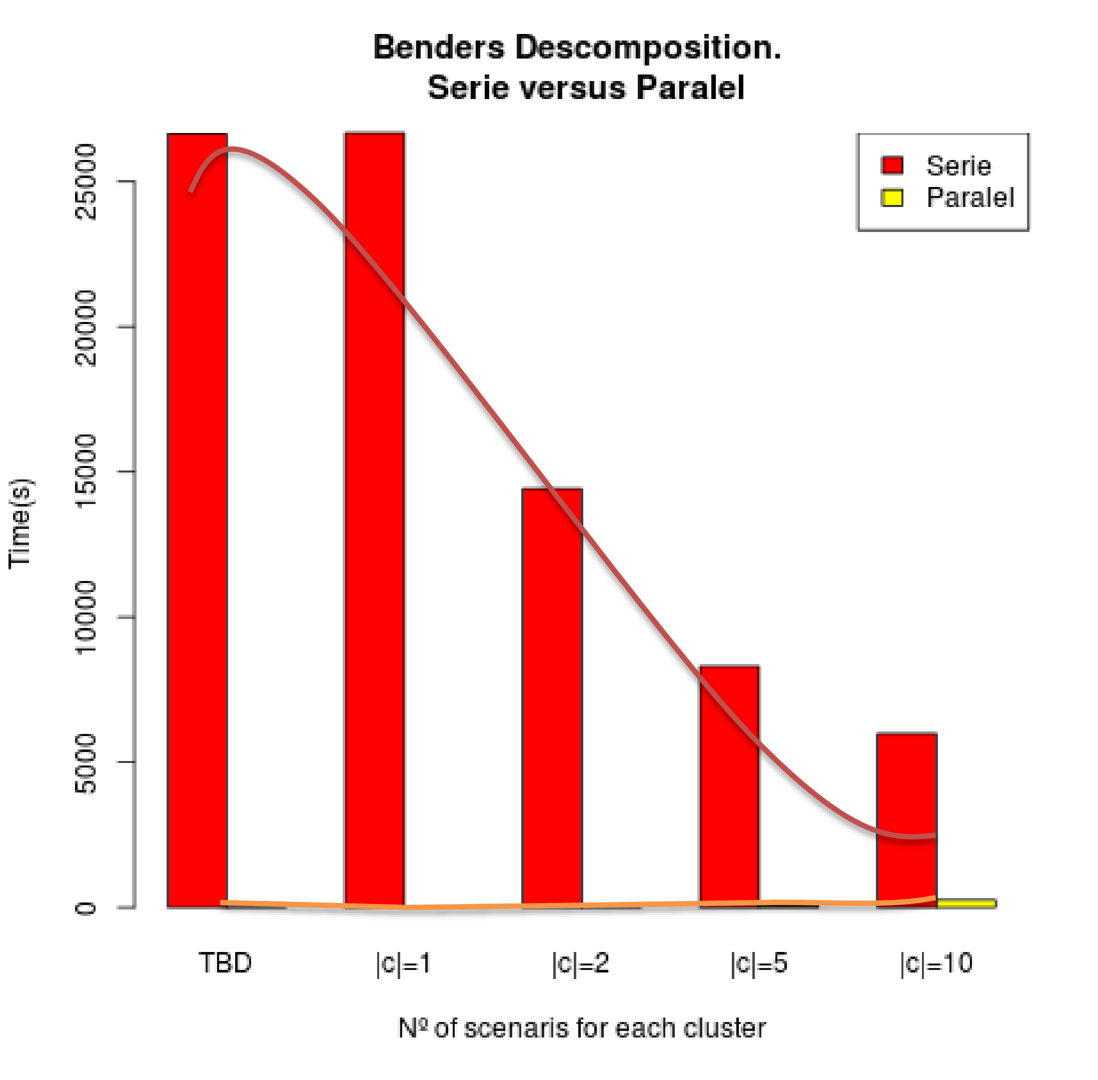
Small Instances

Problema	nc	nv	n_x	n_y	$\|\Omega\|$
P1	1.200	660	60	60	10
P2	2.000	1.100	100	100	10
P3	8.000	4.040	40	40	100
P4	10.000	5.050	50	50	100
P5	12.000	6.060	60	60	100
P6	16.000	8.040	40	40	100
P7	20.000	10.050	50	50	200
P8	24.000	12.060	60	60	200
P9	32.000	16.040	40	40	400
10	40.000	20.040	40	40	500
P11	50.000	25.050	50	50	500

Instance P10



| C |	Time (s)
1	26.684
2	14.421,6
5	8.306,96
10	5.983,17

| C |	Time (s)
1	22,74
2	31,66
5	68,3
10	251,2

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Medium Instances

Problema	nc	nv	n_x	n_y	$\| \Omega \|$
P14	250.000	200.100	100	400	500
P15	250.000	125.250	250	250	500
P16	500.000	450.100	100	900	500

Performance

An ⋀ increase c

\+

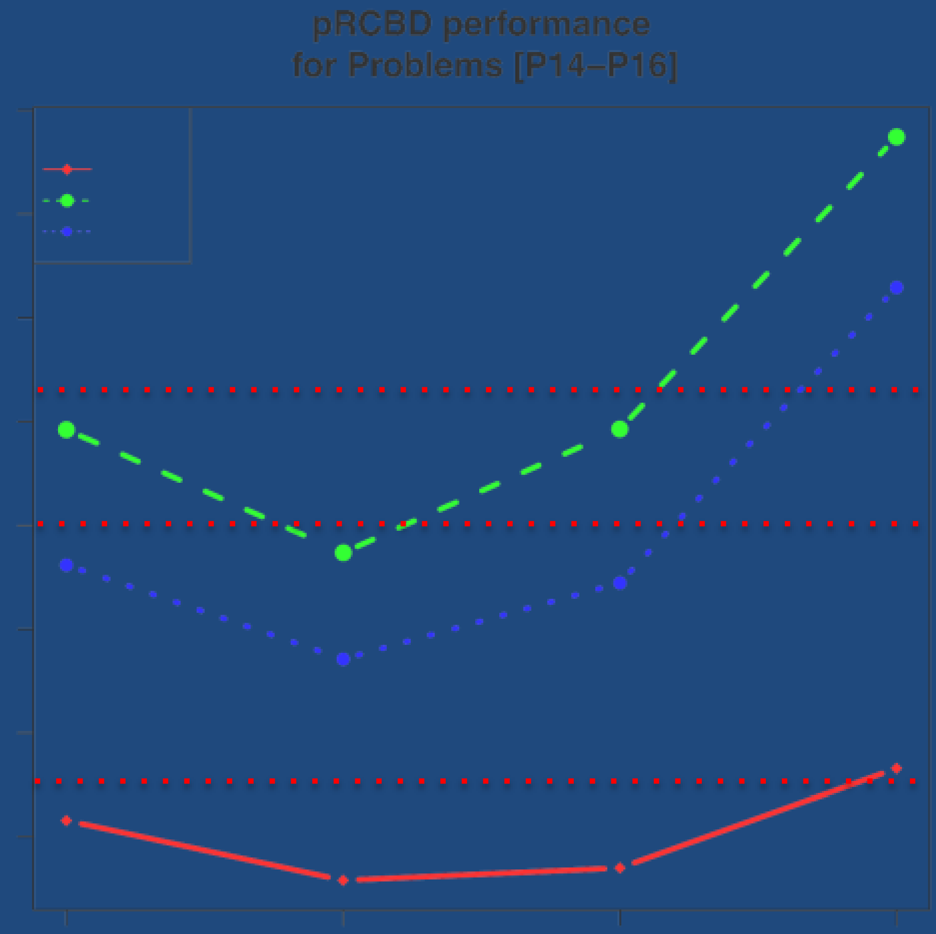
An ⋁ master it

\+

An ⋀ time fworker

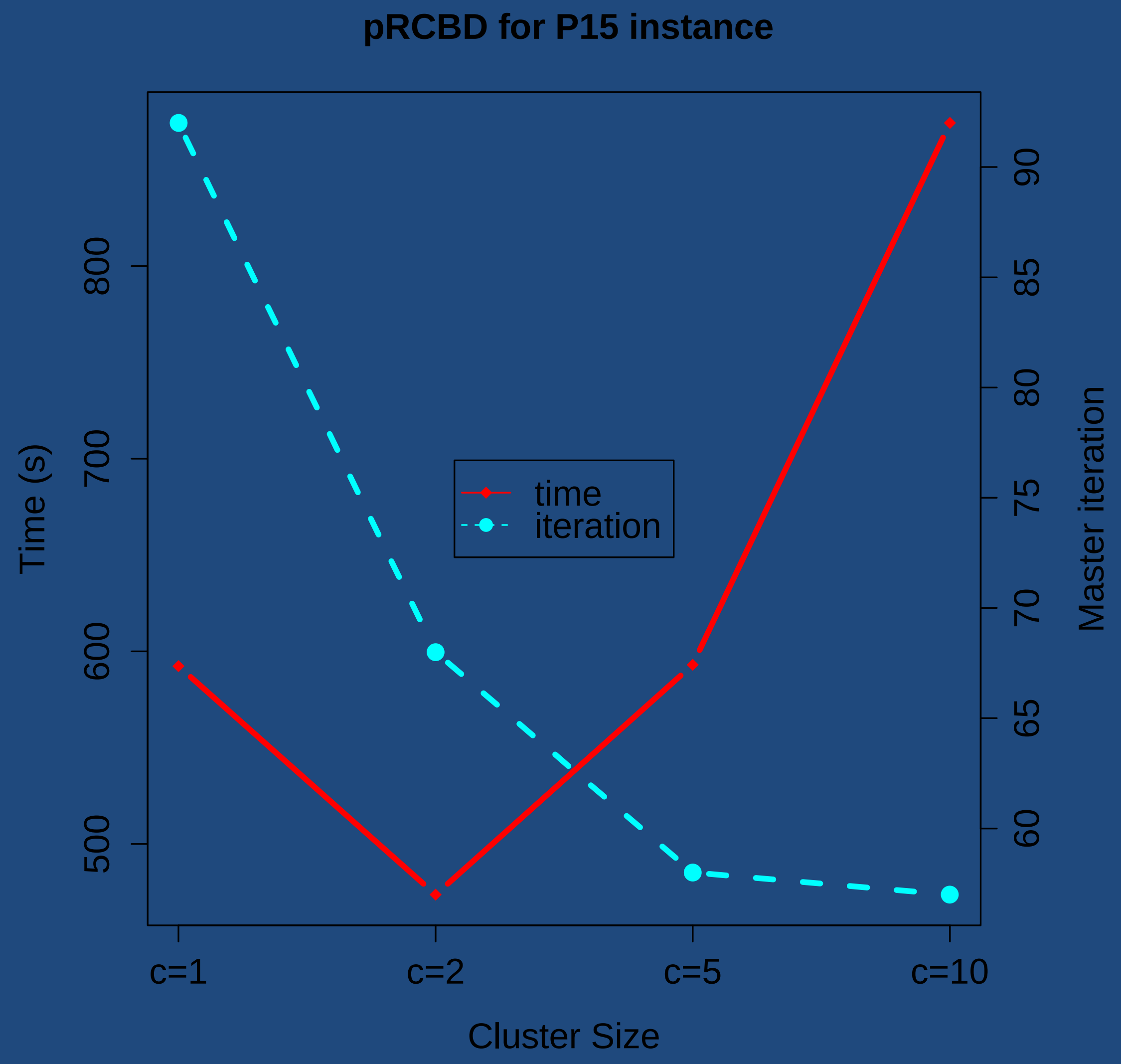
↓

V shape



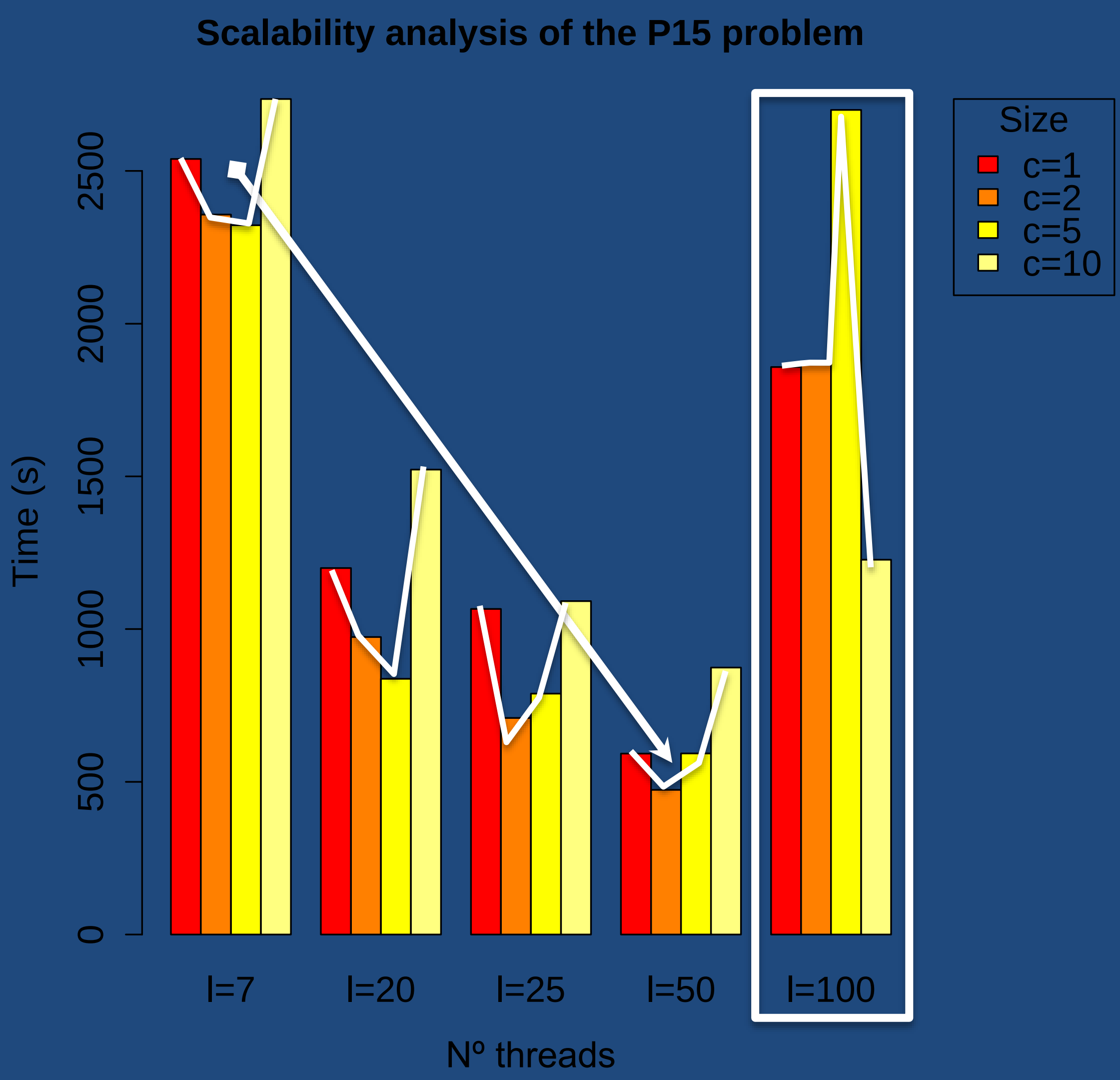
Instance P15

V Shape



← master it

Scalability



Decremental trend

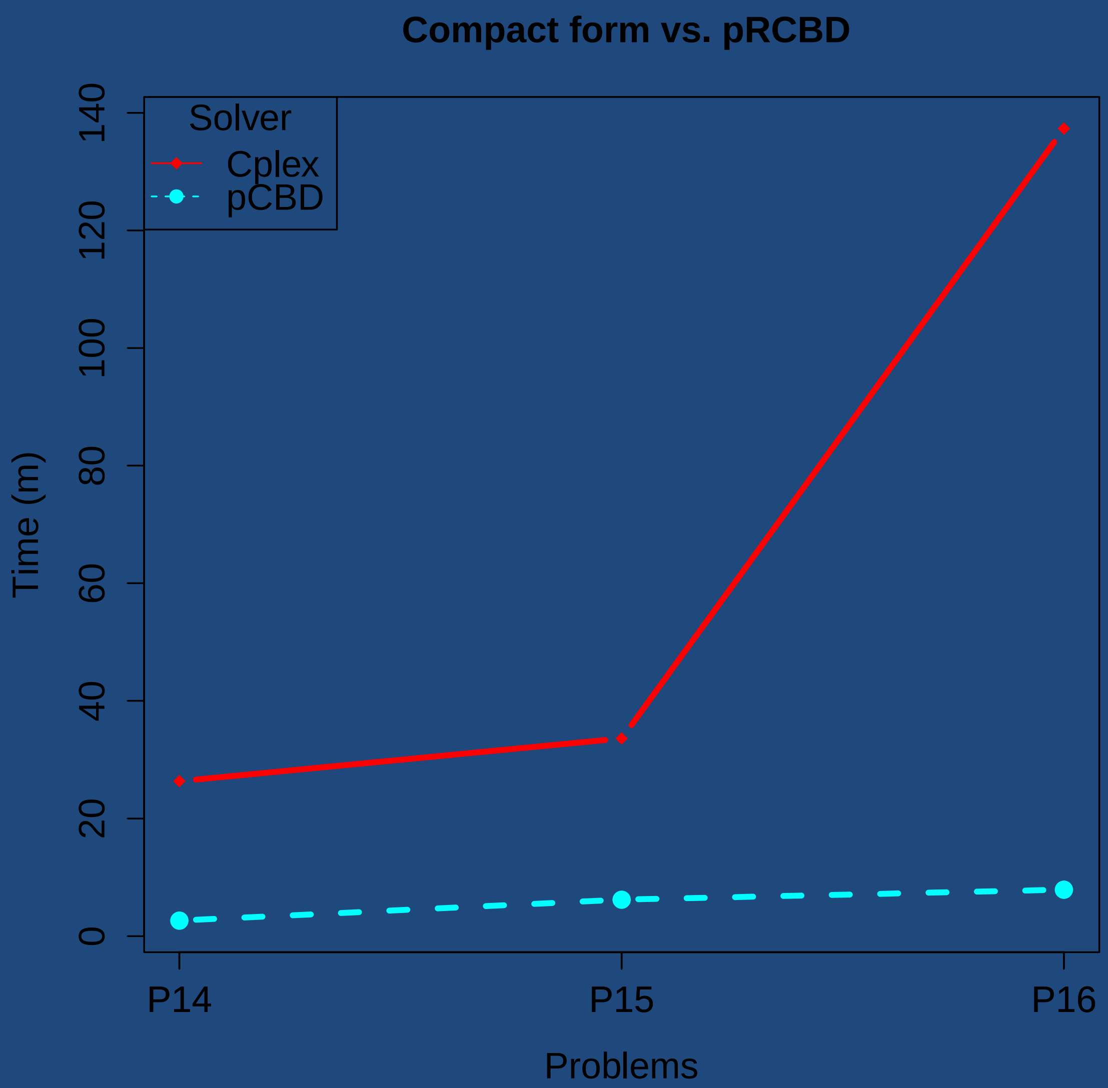
V Shape's

Saturation Point

|||

Good scalability

Compact form vs. PCBD



Good Resolution Times

Applicability

Efficiency

Problems Type

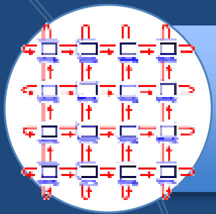
Now

- ✔ Lineal
- ✔ Stochastic
- ✔ Continuous Variables
- ✔ Two-Stage
- ✔ Compact Representation

Future

- ✘ Mix Problems
- ✘ Integer Variables
- ✘ Multi-Stage
- ✘ General Problems

Future Research


Parallel Nested Benders for Multi Stage Problems.


Esteve's Pigs Models.


Mix cloud and parallel computing with optimization methods.


Develop a farmers tool.

Schema

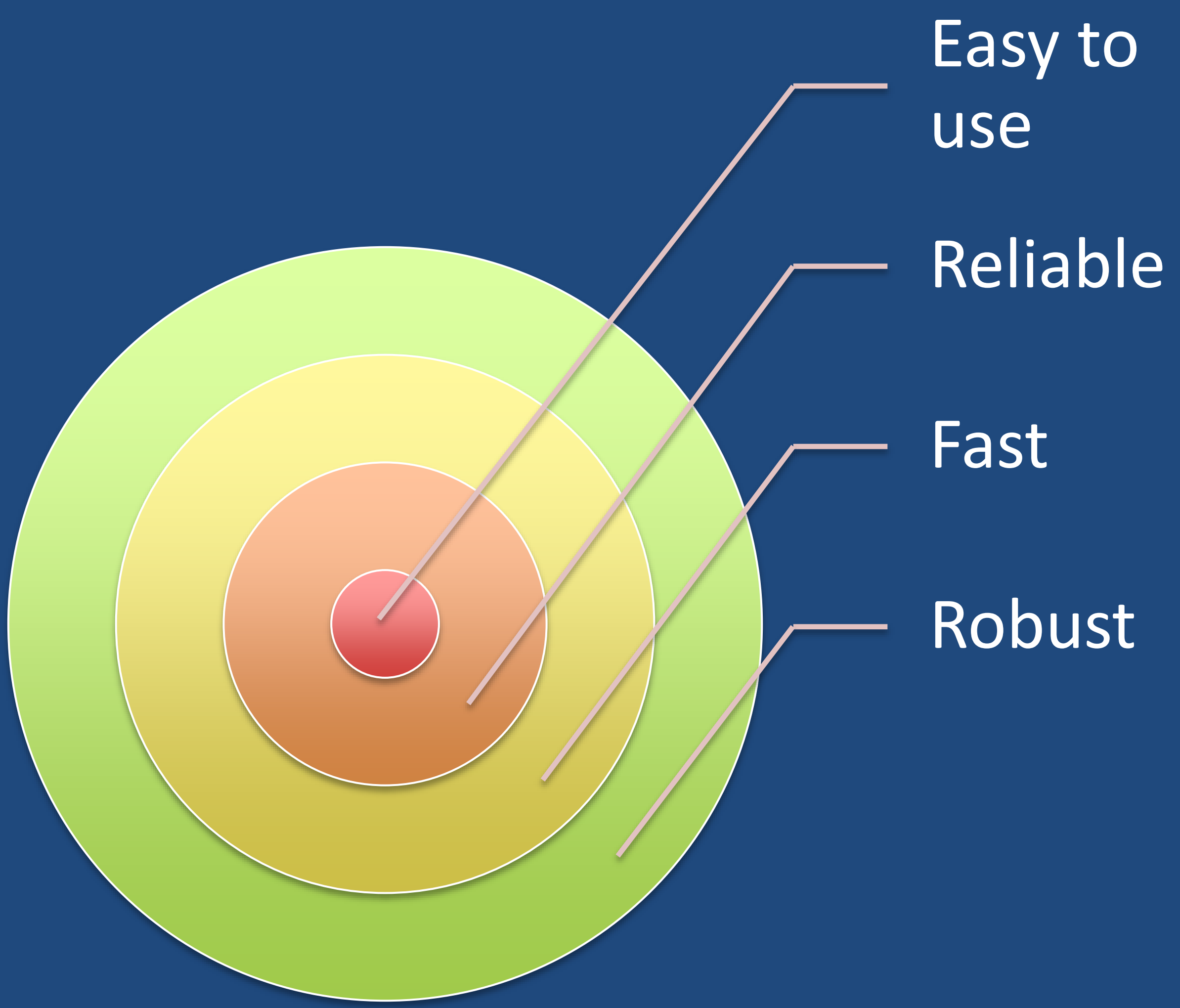
Users

Improve Lleida's economy.
Improve pigs farms.

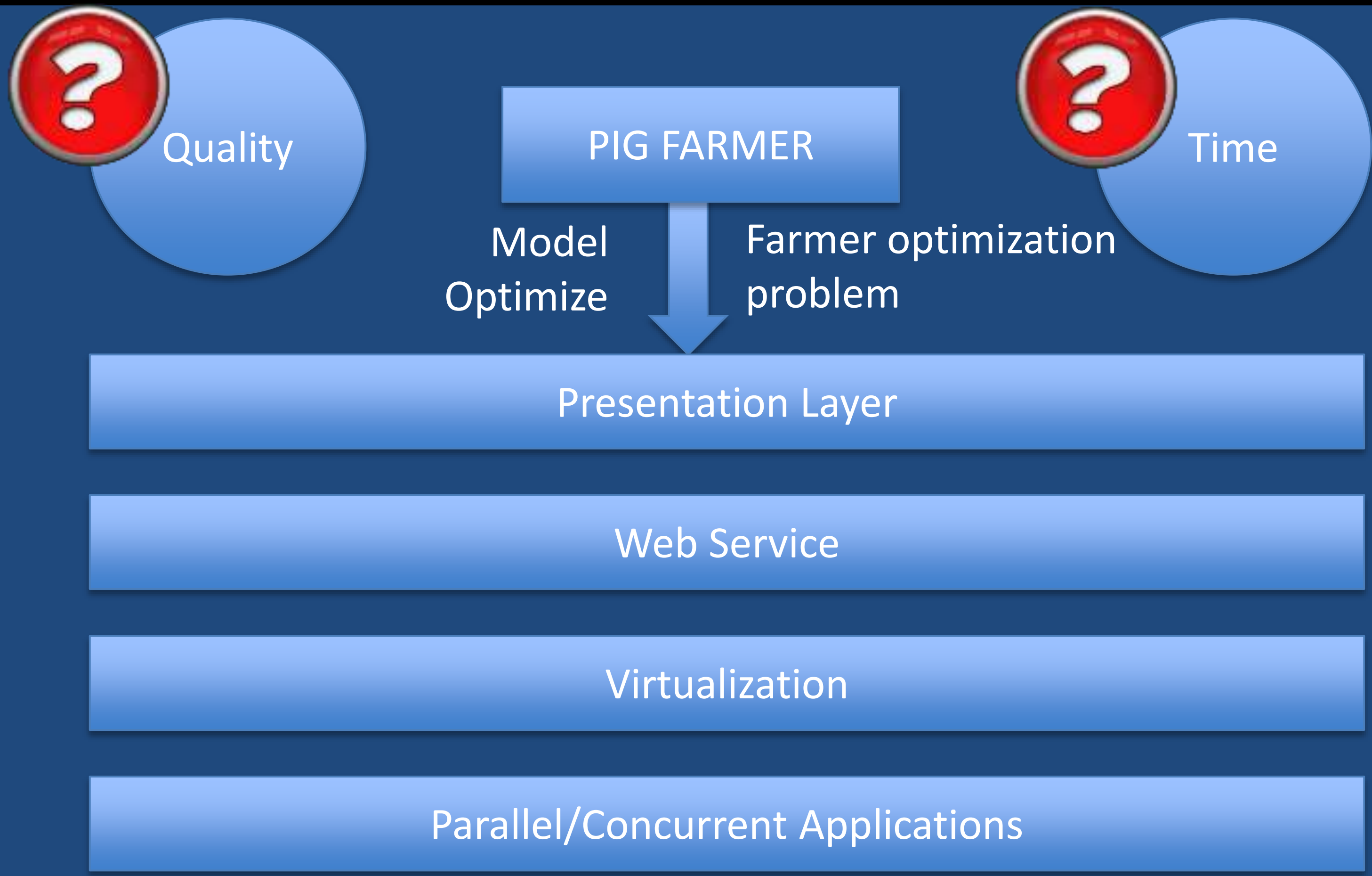
Model

Optimization

Characteristics



Framework


Quality
PIG FARMER
Time
Model
Optimize
Farmer optimization
problem
Presentation Layer
Web Service
Virtualization
Parallel/Concurrent Applications

References

Benders

Introduction to Stochastic Programming, second ed. Springer, New York. 2011

Cluster Benders

A so-called Cluster Benders Decomposition approach for solving two-stage stochastic linear problems. TOP 20, 279-295

Parallel Benders

Performance Optimization of a Parallel, Two Stage Stochastic Linear Program.

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QUESTIONS?