DEPARTMENT OF ECONOMICS AND BUSINESS
AARHUS UNIVERSITY

A hierarchical Markov decision process modelling feeding and marketing decisions of growing pigs

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Agenda

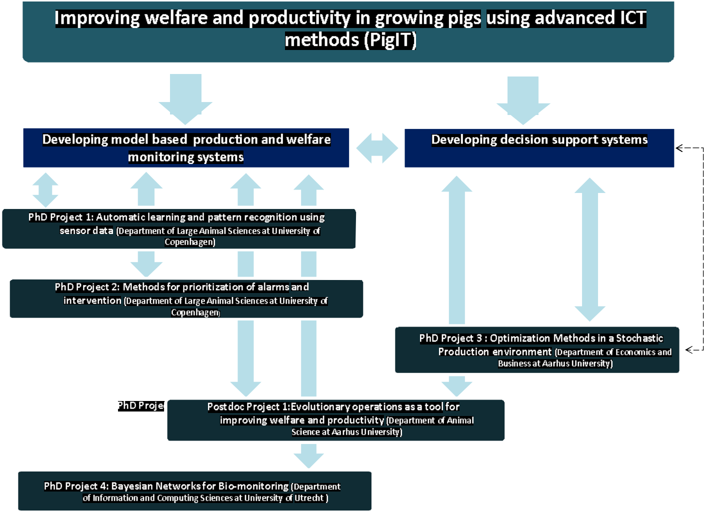
PigIT research group

Optimal feeding and marketing decisions

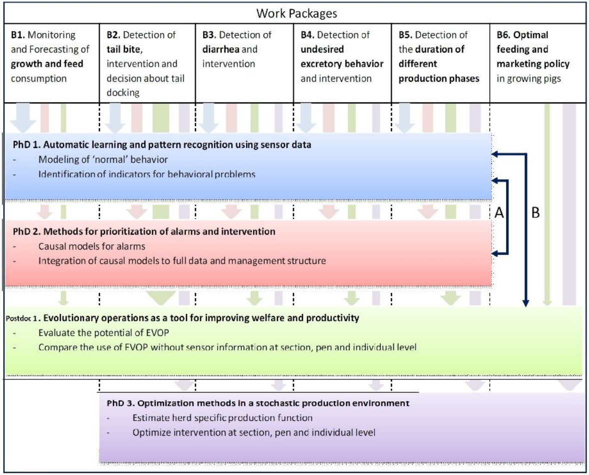
- Problem definition
- Modelling approach
- Initial results

Further research

PigIT project



Work packages



Source: http://pigit.ku.dk/

Agenda

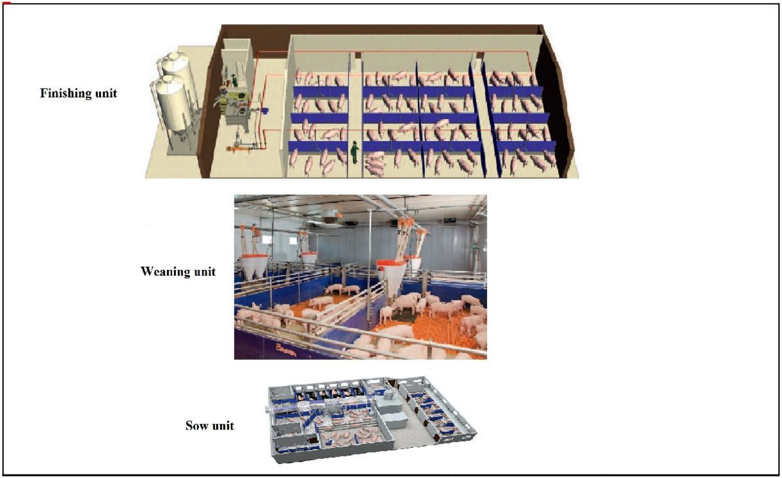
A sub-project in the work package B6

A hierarchical Markov decision process modelling feeding and marketing decisions of growing pigs

Contributions:

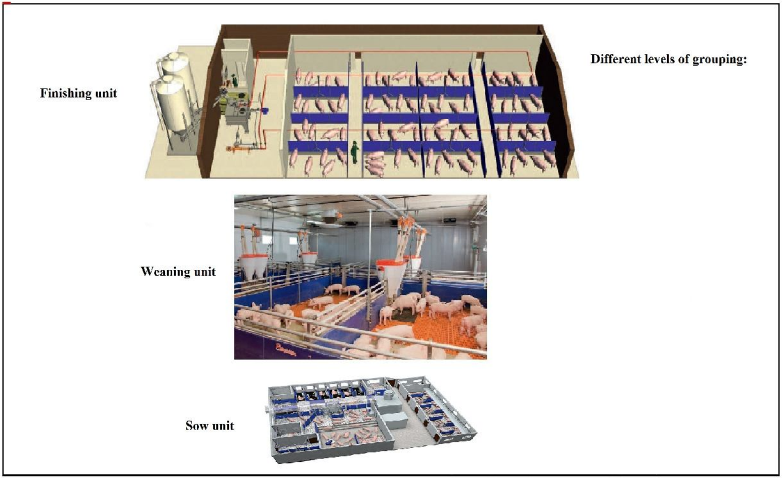
- Combination of feeding and marketing decisions by a hierarchical Markov decision process (HMDP).
- Embedding a dynamic linear model (DLM) and a dynamic generalised linear model (DGLM) into the proposed HMDP.

Problem definition



Source: http://www.bigdutchman.de/

Problem definition



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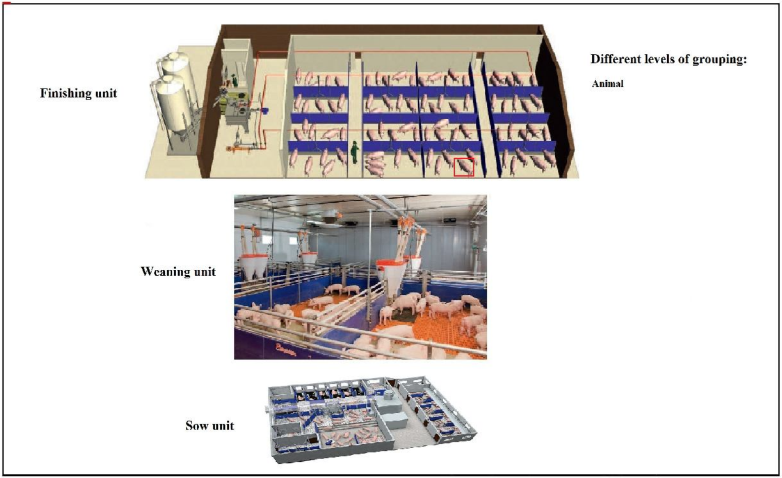

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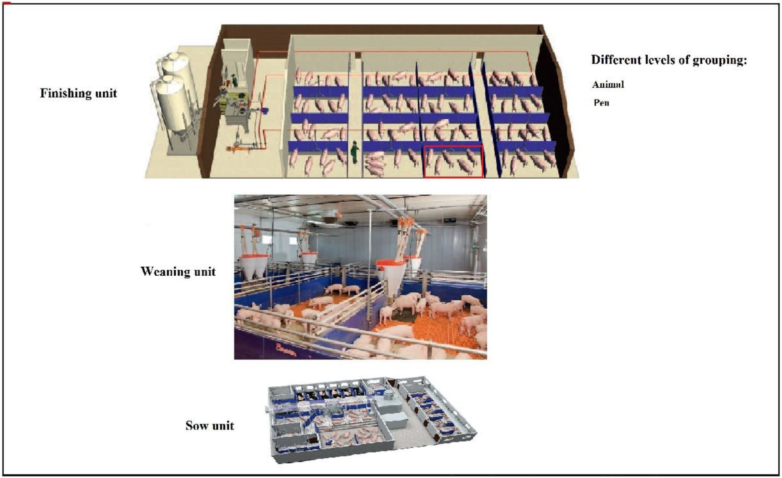

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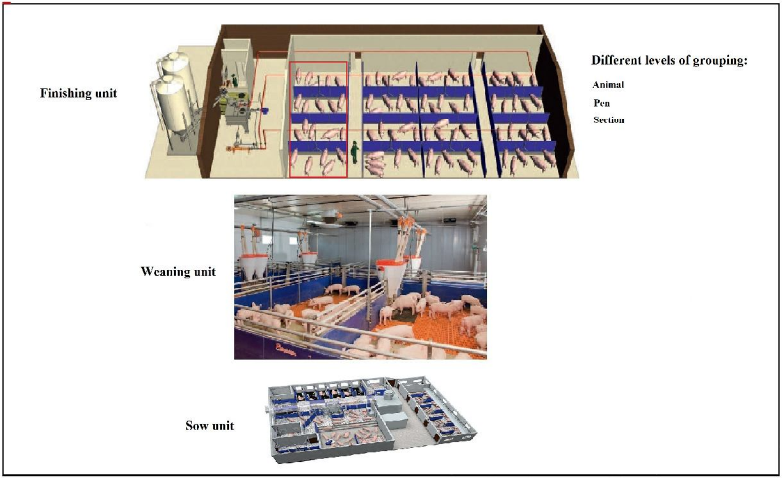

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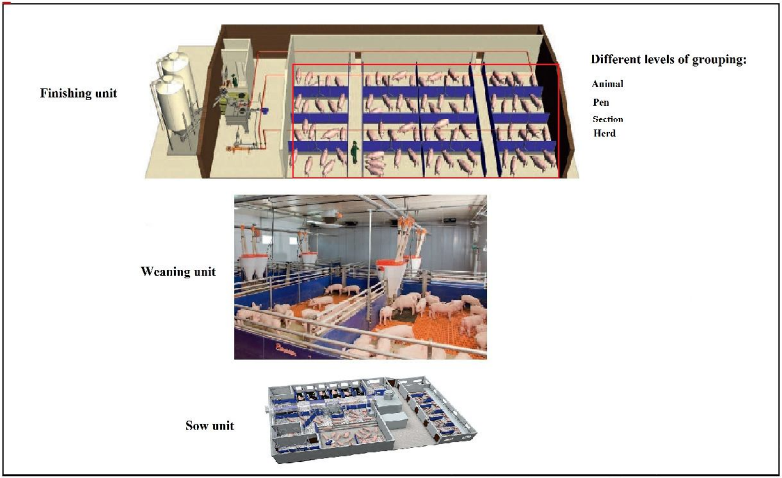

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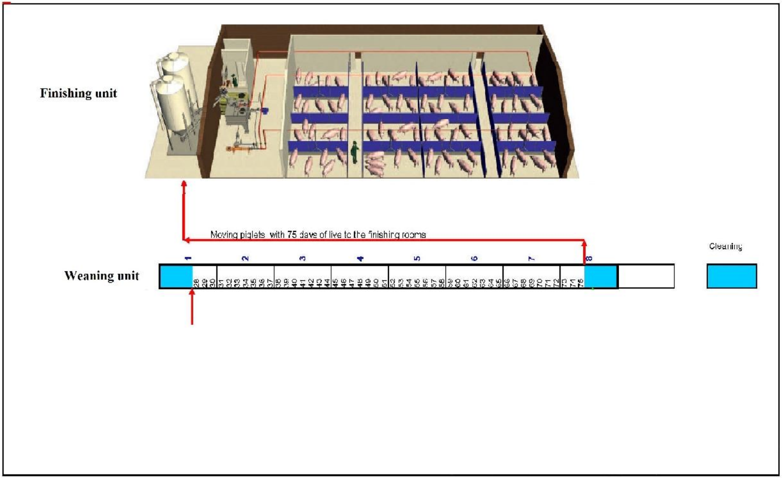

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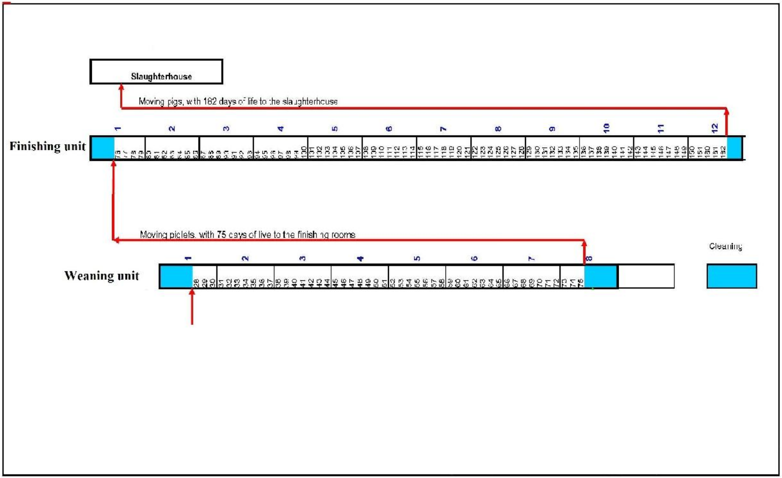

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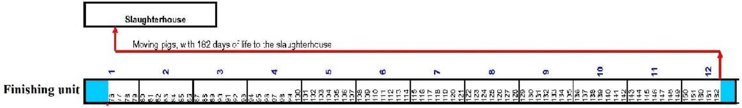
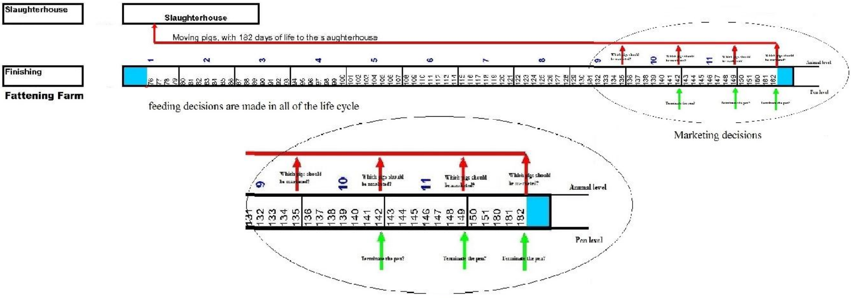


We focus on marketing and feeding decisions in the finishing unit

Source: http://www.bigdutchman.de/

Problem definition - Marketing decisions

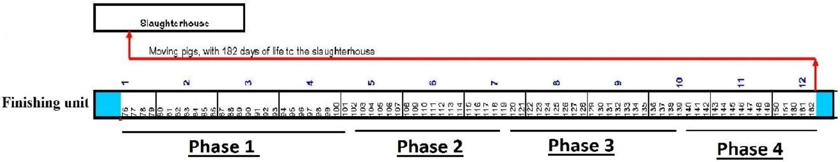


Source: http://www.bigdutchman.de

- Pen (or section) level: decision about when to terminate a pen and insert a new group of piglets to the pen.
- Individual (Animal) level: decisions about how to select and when to market individual animals for slaughter.

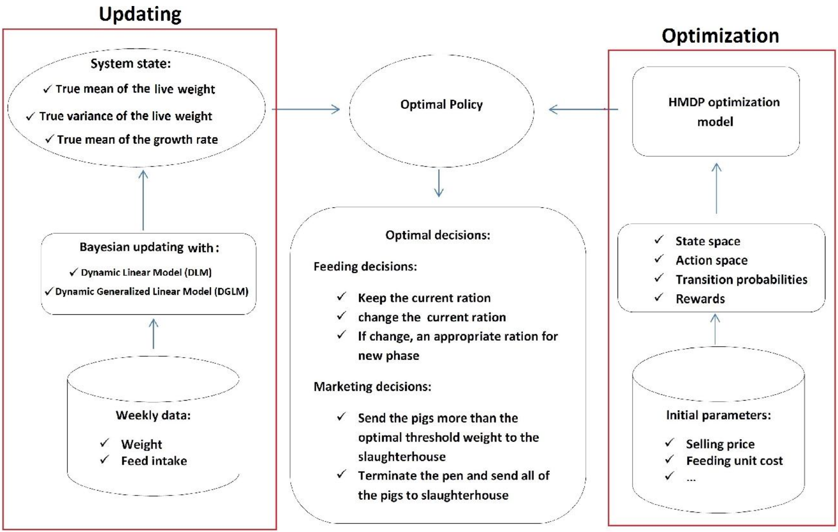
Problem definition - Feeding decisions

Phase feeding



- Decision about the time that the current ration (in the current phase) should be changed.
- Decision about finding the best new ration in the next phase.

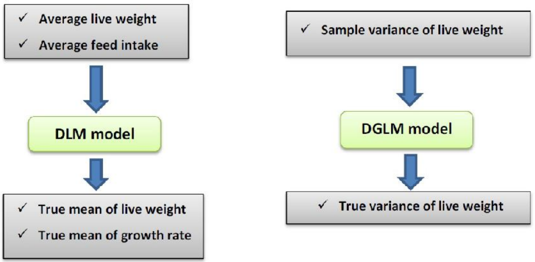
Modelling approach - framework (pen level)



Updating model - Observations

The observations in the pen for each week (or day):

- Average live weight
- Sample variance of the live weight
- Average feed intake



Updating model - DLM

- DLM is a statistical model to analyse the time series and extract the true trend of latent variables (θ_t) based on the observed data:

- A system equation describes the extension of the latent variable against time t:

$$\theta_t = G\theta_{t-1} + \omega_t, \quad \omega_t \sim N(0, V_t)$$

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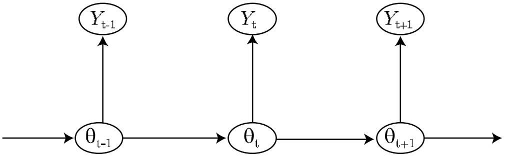

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- An observation equation indicates the relation between the observable (Y_t) and latent (θ_t) variables:

$$Y_t = F'\theta_t + \nu_t, \quad \nu_t \sim N(0, W_t)$$

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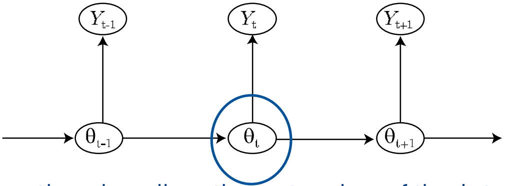

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Updating model - DLM

Observation equation: $$\begin{pmatrix} W_{t,f} \\ FE_{t,f} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ k_{1(t,f)} & k_2 \end{pmatrix} \begin{pmatrix} TW_{t,f} \\ Z_{t,f} \end{pmatrix} + v$$

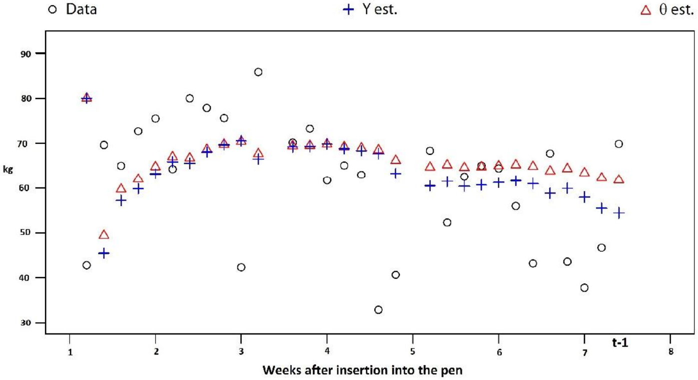
System equation: $$\begin{pmatrix} TW_{t,f} \\ Z_{t,f} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} TW_{t-1,f} \\ Z_{t-1,f} \end{pmatrix} + \omega$$

- $W_{t,f}$: Average live weight at time t
- $FE_{t,f}$: Average feed intake in interval $[t, t+1]$
- $TW_{t,f}$: True live weight at week t
- $Z_{t,f}$: True growth rate at time t
- V : System variances (constant) - estimated by EM algorithm
- W : Observation variances (constant) - estimated by EM algorithm

Updating model - Applying Bayesian updating in the DLM

Data up to time $t-1$: D_{t-1}

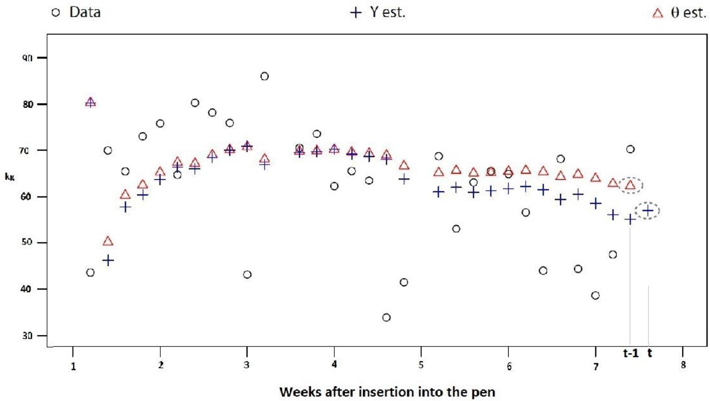
Posterior at $t-1$: $(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$



Updating model - Applying Bayesian updating in the DLM

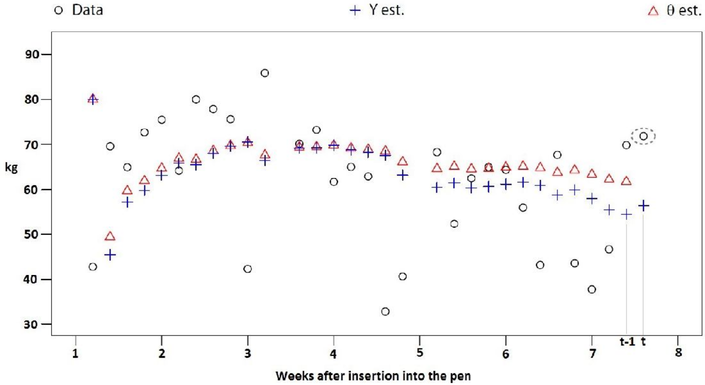
Given $(\theta_{t-1} \mid D_{t-1}) \sim N(m_{t-1}, C_{t-1})$ we have that

$$(Y_t \mid D_{t-1}) \sim N(f_t, Q_t) \text{ where, } f_t = F_t' G_t m_{t-1}$$



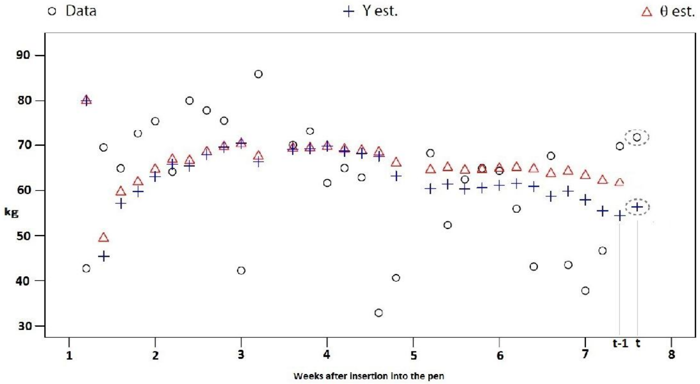
Updating model - Applying Bayesian updating in the DLM

At time t we see an observation y_t:



Updating model - Applying Bayesian updating in the DLM

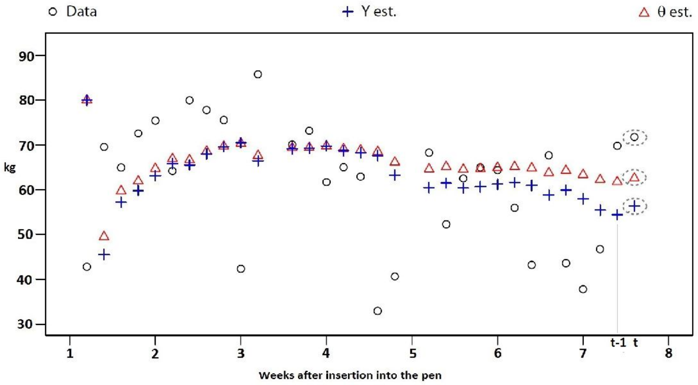
Given f_t and y_t we have that



Updating model - Applying Bayesian updating in the DLM

Given f_t and y_t we have that

$$(\theta_t \mid D_t) \sim N(m_t, C_t) \text{ where, } m_t = G_t m_{t-1} + R_t F_t Q_t (y_t - f_t)$$



Updating model - DGLM

Observation equations:

$$f(s_t^2|\eta_t) = \frac{exp(\frac{-s_t^2(n_t-1)}{2\sigma_t^2})(\frac{n_t-1}{2\sigma_t^2})^{\frac{n_t-1}{2}}(s_t^2)^{\frac{n_t-3}{2}}}{\Gamma(n_t-1)}$$

$$g(\eta_t) = \sigma_t^2$$

where $g(x) = \frac{1}{x}$

System equations:

$$\sigma_t^2 = G_t\sigma_{t-1}^2$$

- s_t^2 : Sample variance of the live weights in pen.
- σ_t^2 : True variance of the live weight in the pen.
- $G_t = \frac{t}{(t-1)}$
- n_t : Number of pigs in the sample for s_t^2

Updating model - Validating the DLM

- To validate the DLM model, the observations were generated by a simulation approach and we supposed the true means are given in this step.
- The filtered data (output of the DLM model) show a good prediction of the true means of the live weight and the true means of the growth rate in the pen.
- Initial results indicate a good learning capability of the suggested DLM model.