Analysis of Pricing and Protection Strategies in a Supply Chain under Uncertain Demand

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ESI 2014

To be published in: *International Journal of Production Economics*

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Introduction

- Consider a two-echelon supply chain comprising a single manufacturer and a single retailer, who distributes the product to end-customers. The retail price of the product is affected by both the manufacturer and the retailer.
- Early studies in the field of supply chain research tended to assume that the market demand for a given product is determined according to a single variable—the retail price.
- Recent studies, on the other hand, have increasingly begun to acknowledge additional factors that may affect demand (for a comprehensive survey of demand functions in decision modeling see Huang, Leng and Parlar (2013)). We analyze two decision variables that affect demand.

Introduction (continue)

- Both parties wish to maximize their own profit. This vertical relationship is usually analyzed by a game approach

- Different types of games reflect different power balances between the players and different sequences of their decisions:
    - a symmetrical power balance yielding a game of simultaneous decisions
    - leadership of either the manufacturer or the retailer with sequential decisions—such scenarios are commonly modeled using Stackelberg games
    - the case of a manufacturer and a retailer who act in cooperation and bargain for the division of profits—this case is reflected in bargaining game approaches

Introduction (continue)

- In reality, demand is a random variable due to various sources of future uncertainty (e.g., seasonality, changes in customers' tastes, introduction of competitive products and technological developments).
- As a result of the stochastic nature of demand, the manufacturer and the retailer are exposed to financial risks, and the two parties make their decisions according to their respective attitudes toward risk.
- We use utility functions to express these attitudes.

Specific Motivation

- Digital products such as software programs, digital music files and videos often implement digital rights management (DRM) systems designed to control how end-users can install, copy, or duplicate these products.
- The positive effect of DRM investment on customer demand is similar to that of other demand accelerators such as rebate, advertising, and quality.
- However, beyond a certain level of protection, DRM systems may decrease the value of the original product, as it is perceived by consumers.

Motivation (continue)

- As is common in practice, the manufacturer alone determines how much to invest in DRM, whereas the retail price of the product is affected by both the manufacturer and the retailer.

- we consider two models:
    - The first model, which is based on the Manufacturer-Stackelberg (MS) game, assumes that the manufacturer is the leader. (For example, Microsoft, a software manufacturer that dominates the market and is much larger than the retailers selling its products).
    - The second model, which is based on the Retailer-Stackelberg (RS) game, assumes that the retailer is the leader. (For example, Apple, a retailer of music and smart-phone applications, sold through the iTunes store, that is much larger and more dominant than most of the content manufacturers).

Literature review

Previous research grouped into 4 categories:

- Literature on pricing and protection strategies of digital products
  - Conner KR, Rumelt RP. (1991). Software piracy: An analysis of protection strategies. *Management Science*, 37, 125-139.
- Utility functions and other profit criteria under uncertainty. The Target criterion, which is the probability that the profit will be no less than a predefined target level, T, is commonly used as a satisficing objective both in the literature and in practice
  - Shi CV, Zhao X, Xia Y. (2010). The setting of profit targets for target oriented divisions. *European Journal of Operational Research*, 206, 86–92.

Literature review (continue)

- Stochastic dominance of distributions.

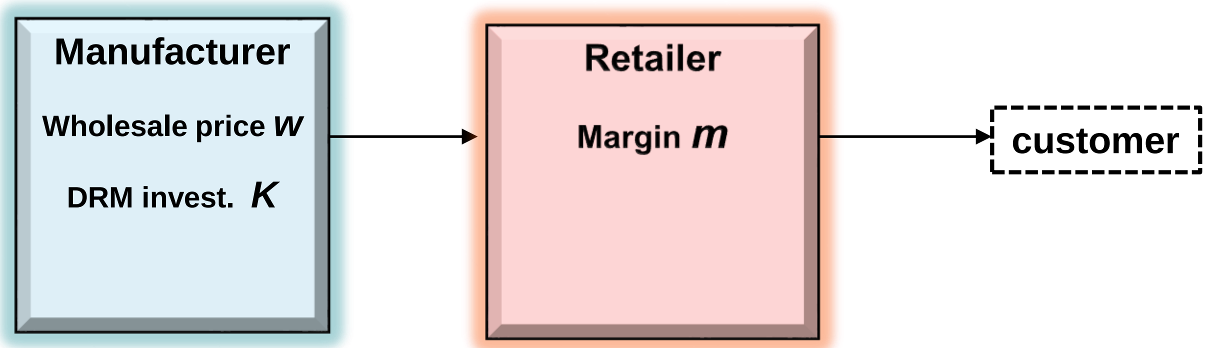
  In general, if a random variable X stochastically dominates a random variable Y, denoted $X \succ Y$ then

  $$P(X \le z) \le P(Y \le z)\ \forall z$$

  - Hadar J, Russell WR. (1971). Stochastic dominance and diversification. *Journal of Economic Theory*, 3, 288–305.

- Structure of the demand function. Two models are common in the literature:
  - The multiplicative-demand model, where a change in one factor has a relative effect on demand for any value of the other factor.
  - The additive-demand model, where a change in one factor has an absolute effect on demand for any value of the other factor

Model formulation



- Retail price $p = m + w$
- Demand $\widetilde{D}(p, K) \equiv D(p, K)\varepsilon$
  - Decreasing in *p*
  - Increasing in K up to κ
- Both parties wish to maximize their own expected utility of the profit
- Profits:
  - Retailer's profit $\tilde{\pi}_R(m) \equiv mD(m + w, K)\varepsilon$
  - Manufacturer's profit $\tilde{\pi}_M(w, K) \equiv wD(m + w, K)\varepsilon$-K

Model (continue)

Theorem 1. The margin m that maximizes $\pi_R(m)$ produces a profit that stochastically dominates the profits that the retailer attains with other margins.

Theorem 2. The wholesale price w that maximizes $\pi_M(w, K)$ for a given K produces a profit that stochastically dominates profits attained with other wholesale prices.

Insights

Theorem 1 claims that the retailer can determine her optimal margin exactly as in a deterministic demand model regardless of her attitude toward risk or the distribution of ε.

Theorem 2 indicates that for a given K, the manufacturer should determine his wholesale price exactly as in a deterministic demand model.

The multiplicative demand model

$$D(p, K) = g(p)h(K)$$

g(p) - the price effect (strictly decreasing)
h(k) - the DRM investment effect (strictly increasing and strictly concave up to κ)

Two models of asymmetric power-balance:
(i) the manufacturer is the leader
(ii) the retailer is the leader
Vertical integrated firm is analyzed as a benchmark

Stackelberg Games →

Insights

Theorem 3. In the MS game, the equilibrium prices $\boldsymbol{m}^{MS}$ and $\boldsymbol{w}^{MS}$ depend only on the price effect on the demand, $g(\cdot)$

Theorem 1 claims that the retailer can determine her optimal margin without having to assess her own utility function, the manufacturer's utility function or the distribution of ε.

Theorem 3 indicates that she can do so even without knowing the DRM investment effect on the demand.

DRM investment decision can be postponed until after the pricing decisions are made.

Insights (continue)

Theorem 4. Under the Target, if $T_2 > T_1$ then:

(i) $\tilde{D}\left(m+w, K_T \middle| T = T_2\right) \succ \tilde{D}\left(m+w, K_T \middle| T = T_1\right)$;

(ii) $\tilde{\pi}_r\left(m \middle| T = T_2\right) \succ \tilde{\pi}_r\left(m \middle| T = T_1\right)$.

The retailer is better off working with a manufacturer who poses a higher target level, because that manufacturer will invest more in DRM, causing the sales volume to be higher.

Insights(continue)

Theorem 5. Under the Target criterion, the following claims hold:

(i) The equilibrium retail prices in the MS and RS games are identical,

$$m_T^{RS} + w_T^{RS} = m^{MS} + w^{MS}.$$

(ii) The equilibrium DRM investments in the two games are identical, $K_T^{RS} = K_T^{MS}$.

(iii) The two games yield identical values for expected sales volume and for total expected profit.

The customer is indifferent between MS/RS under the Target criterion.

Linear-square-root demand model

$$g(p) = a - bp, h(k) = \sqrt{K}, p_{max} \equiv \frac{a}{b}, \theta \equiv \frac{a^2}{b}$$

Results under the Expectation criterion

Results under the Target criterion

Results under a non-monotonic effect of DRM investment

Summary

- When demand is uncertain, attitudes toward risk are crucial to decision making. One of the main conclusions of this paper is that the analysis of such a case is possible. In certain stochastic models, as shown above, stochastic dominance prevails, and the optimization process is similar to that associated with a deterministic demand.

- Under the linear square root model, the maximum expected profit of the manufacturer is higher in the RS game than in the MS game. A managerial implication of this result is that there are situations in which the manufacturer is better off giving up his leadership even if the power balance is in his favor. This might explain the recent emergence of dominant retailers in digital product supply chains (e.g., iTunes, Google Play).

- We find that when customers are highly tolerant of the negative aspects of DRM (i.e., when the threshold is very high), not only do the retailer and the manufacturer benefit, but the customers themselves benefit as well i.e., they enjoy lower prices.

End

Manufacturer Stackelberg

Step 1: Find the best retailer response $m(w)$ that maximizes $mg(m+w)$.

Step 2: Find the equilibrium wholesale price w^{MS} that maximizes $wg(m(w)+w)$.

Step 3: Find the equilibrium DRM investment K^{MS} that maximizes $E\{u(w^{MS}g(m^{MS}+w^{MS})h(K)\varepsilon - K)\}$ where $m^{MS} \equiv m(w^{MS})$ is the equilibrium margin.

Step 3 is simplified under the Expectation criterion: K^{MS} maximizes $w^{MS}g(m^{MS}+w^{MS})h(K)-K$; the solution is denoted K_E^{MS}.

Retailer Stackelberg

Step 1: Find the best manufacturer response $w(m)$ that maximizes $wg(m+w)$.

Step 2: Find the best manufacturer response $K(m)$ that maximizes

$E\left\{u\left(w(m)g(m+w(m))h(K)\varepsilon - K\right)\right\}$.

Step 3: Find the retailer's equilibrium margin m^{RS} that maximizes $mg(m+w(m))h(K(m))$.

In Step 2, under the Expectation criterion, $K(m)$ maximizes $w(m)g(m+w(m))h(K)-K$; and under the Target criterion, $K(m)$ maximizes $P(\tilde{\pi}_M(w(m),K)\geq T)=1-F_\varepsilon\left(\dfrac{T+K}{w(m)g(m+w(m))h(K)}\right)$; this solution is attained by minimizing $(T+K)/h(K)$.

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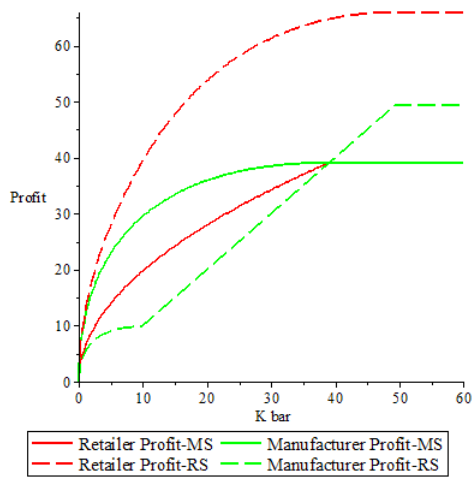
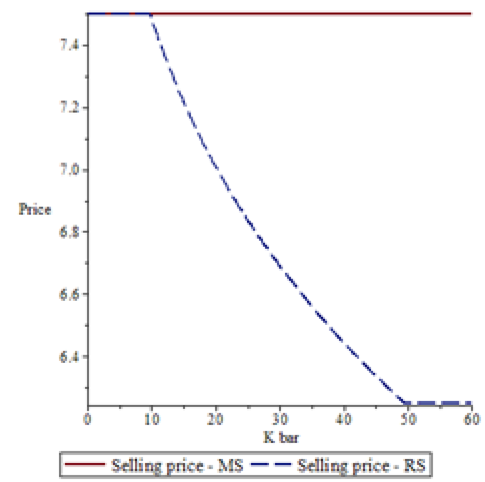
Results under the Expectation criterion

Variable	MS		RS		
	$\kappa < 0.0039\theta^2$	$\kappa \geq 0.0039\theta^2$	$\kappa < 0.001\theta^2$	$0.001\theta^2 \leq \kappa < 0.0049\theta^2$	$\kappa \geq 0.0049\theta^2$
p	$0.75p_{\max}$	$0.75p_{\max}$	$0.75p_{\max}$	$p_{\max} - (4\kappa/b^2)^{0.25}$	$0.625p_{\max}$
m	$0.25p_{\max}$	$0.25p_{\max}$	$0.25p_{\max}$	$p_{\max} - 2(4\kappa/b^2)^{0.25}$	$0.25p_{\max}$
w	$0.5p_{\max}$	$0.5p_{\max}$	$0.5p_{\max}$	$(4\kappa/b^2)^{0.25}$	$0.375p_{\max}$
K	κ	$0.0039\theta^2$	κ	κ	$0.0049\theta^2$
D	$0.25a\kappa^{0.5}$	$0.0156a\theta$	$0.25a\kappa^{0.5}$	$(4\,b^2\kappa^3)^{0.25}$	$0.0264a\theta$
π_R	$0.0625\theta\kappa^{0.5}$	$0.0039\theta^2$	$0.125\theta\kappa^{0.5}$	$(4\theta^2\kappa^3)^{0.25} - 4\kappa$	$0.0066\theta^2$
π_M	$0.125\theta\kappa^{0.5} - \kappa$	$0.0039\theta^2$	$0.0625\theta\kappa^{0.5} - \kappa$	κ	$0.0049\theta^2$
$\pi_R + \pi_M$	$0.1875\theta\kappa^{0.5} - \kappa$	$0.0078\theta^2$	$0.1875\theta\kappa^{0.5} - \kappa$	$(4\theta^2\kappa^3)^{0.25} - 3\kappa$	$0.0115\theta^2$





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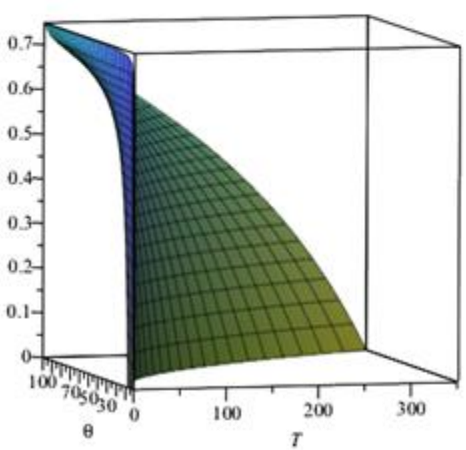
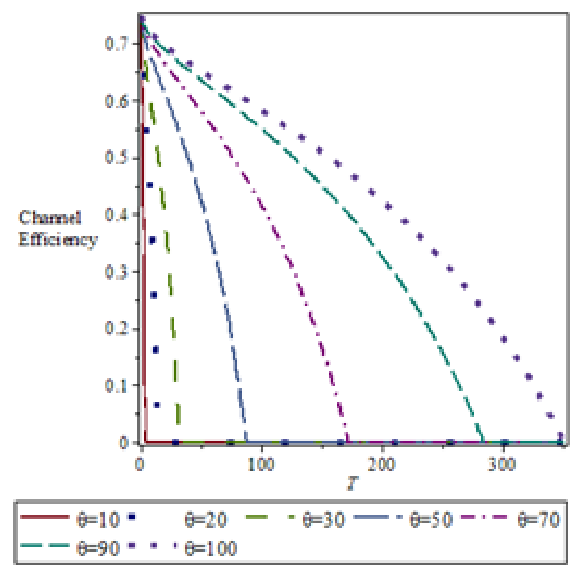
Results under the Target criterion

Model / Variable	MS	RS	VI
p	$0.75p_{\max}$		$0.5p_{\max}$
m	$0.25p_{\max}$	$0.5p_{\max}$	
w	$0.5p_{\max}$	$0.25p_{\max}$	
K	T		
D	$0.25a\sqrt{T}$		$0.5a\sqrt{T}$
π_R	$0.0625\theta\sqrt{T}$	$0.125\theta\sqrt{T}$	
π_M	$0.125\theta\sqrt{T}-T$	$0.0625\theta\sqrt{T}-T$	
$\pi_R+\pi_M$	$0.1875\theta\sqrt{T}-T$		$0.25\theta\sqrt{T}-T$





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Equilibrium results under the Expectation criterion

Model / Variable	MS	RS	VI
p	$0.75p_{\max}$	$0.625p_{\max}$	$0.5p_{\max}$
m	$0.25p_{\max}$	$0.25p_{\max}$	
w	$0.5p_{\max}$	$0.375p_{\max}$	
K	$0.0039\theta^2$	$0.0049\theta^2$	$0.0156\theta^2$
D	$0.0156a\theta$	$0.0264a\theta$	$0.0625a\theta$
π_R	$0.0039\theta^2$	$0.0066\theta^2$	
π_M	$0.0039\theta^2$	$0.0049\theta^2$	
$\pi_R + \pi_M$	$0.0078\theta^2$	$0.0115\theta^2$	$0.0156\theta^2$

$p_E^* < p_E^{RS} < p_E^{MS}$

$K_E^* > K_E^{RS} > K_E^{MS}$

$\pi_R^{RS} > \pi_R^{MS}$

$\pi_M^{RS} > \pi_M^{MS}$

Channel Efficiency =0.5

Channel Efficiency =0.737

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